Is Software Eating the World?*

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Abstract

When explaining the declining labor income share in advanced economies, the macro literature finds that the elasticity of substitution between capital and labor is greater than one. However, the vast majority of micro-level estimates shows that capital and labor are *complements* (elasticity less than one). Using firm- and establishment-level data from Korea, we divide capital into equipment and software, as they may interact with labor in different ways. Our estimation shows that equipment and labor are complements (elasticity 0.6), consistent with other micro-level estimates, but software and labor are substitutes (1.6), a novel finding that helps reconcile the macro vs. micro elasticity discord. As the quality of software improves, the labor share falls within firms because of factor substitution and endogenously rising markup. In addition, production reallocates toward firms that use software more intensively, as they become effectively more productive. Because in the data these firms have higher markups and lower labor shares, the reallocation further raises the aggregate markup and reduces the aggregate labor share. The rise of software accounts for two-thirds of the labor share decline in Korea between 1990 and 2018. The factor substitution and the markup channels are equally important.

Keywords: Labor income share, markup, elasticity of substitution, software-embodied technological change, reallocation

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Software is eating the world.

— Marc Andreessen (The Wall Street Journal, August 20, 2011)

1 Introduction

The labor share remained more or less constant for much of the 20th century. For example, Keynes (1939) wrote that "the stability of the proportion of the national dividend accruing to labour" was "one of the most surprising, yet best-established, facts." However, the labor share started a downward trend in the United States and other advanced economies since the 1980s. With the heightened interest in economic inequality after the financial crisis of 2007–08, economists have raised a variety of explanations for the decline of the labor share, as reviewed by Aum and Shin (2020) and Grossman and Oberfield (2022).

One of the leading explanations is that capital and labor are substitutes, and the more efficient production of capital goods reduced the labor share over time (Karabarbounis and Neiman, 2013). This explanation however is at odds with the majority of micro-level estimates that find capital (specifically equipment) and labor are actually complements, with an elasticity of substitution less than one (Antras, 2004; Raval, 2019; Oberfield and Raval, 2021, among others).

We address this macro vs. micro elasticity discord by dividing capital into equipment and software, as their interactions with labor may differ. We estimate the elasticity of substitution across software, equipment, and labor, both at the micro and the macro levels. We then quantify the distinct role of software and equipment price changes (or software/equipment-embodied technological changes) in driving the decline of the labor share.

The macro elasticity differs from the micro elasticity, because changes in factor prices not only alter the factor income shares within firms, but also reallocate resources across firms that are heterogeneous in factor intensities (Houthakker, 1955; Jones, 2005; Oberfield and Raval, 2021). The estimation of the macro elasticity therefore requires micro-level data capturing this cross-sectional distribution of factor shares. More important, for the purpose of separating the elasticity of substitution between software and labor from that between equipment and labor, one needs micro-level data by capital types. Typically, software is lumped together with other intangibles, a category with immense measurement problems.

We use firm-level and plant-level data from Korea, where firms keep track of their software investment to comply with local accounting standards, separately from other categories of intangibles that are more prone to missing values and other measurement problems.

There are reasons to think that software deserves separate investigation. For one, the evolution of the software investment share in the aggregate time series in the US aligns with the dynamics of the labor share, whereas the equipment share does not (Aum and Shin, 2020). For another, software and equipment usage varies across occupations in the US data, with software being primarily used by high-skill occupations and equipment by middle-skill ones (Aum, 2020), which suggests that the two interact with labor in different ways.

We start with two empirical patterns in the Korean data that further validate the case for investigating software separately from equipment. First, firm-level panel regressions show that firms' software intensity predicts a decrease in the labor share, especially the income share of non-production, white-collar workers. In contrast, firms' equipment intensity makes no meaningful prediction either way. Second, in a cross-section of regions, software expenditure shares and local wages are negatively related, but equipment shares and local wages do not have a significant relationship.

To estimate the macro elasticity of substitution between labor and the two types of capital, we build on the approach of Oberfield and Raval (2021). Our contribution is to allow for three factors (labor, equipment, and software) and variable markups, which leads to two novel theoretical results. First, in terms of factor substitution, the role of reallocation depends on the covariance between factor shares across firms, in addition to the variance of factor shares. This is because, unlike in a two-factor model, a high software share of a firm does not necessarily imply a low labor share—it could be that its equipment share is very low and the labor share is also high. Second, variable markups introduce an additional margin of adjustment in the labor share both through within-firm and between-firm adjustments in markups in response to factor price changes.

Using this theoretical framework, we first estimate the micro elasticity of substitution between labor and either type of capital by instrumenting for wage variations across regions or industries. We find that the elasticity of substitution between labor and equipment is less than one (0.6), consistent with the micro-level estimates from the US and other countries. The novel finding is that the elasticity of substitution between labor and software is greater than one (1.6). We obtain very similar estimates with firm-level and plant-level data, and with regional shift-share instruments and industry-level minimum-wage instruments. Because software and labor are substitutes at the micro level, a fall in software price reduces the labor share within a firm.

The effect is aimplified by the variable markups, as firms charge higher markups in response, further reducing labor shares within firms.

The firms using software more intensively benefit more from the fall in software prices. They become effectively more productive and hence larger than those using less software. Using the joint distribution of factor shares and markups from the micro data, we calculate the macro elasticity, accounting for such reallocation across firms. In the data, firms with high software shares tend to have lower-than-average labor shares. (Again, with three factors, they need not.) Hence the reallocation further reduces the aggregate labor share. Also in the data, high software share firms tend to have higher markups. The implied low demand elasticity dampens the magnitude of the reallocation (relative to a constant markup environment). Still, the reallocation to high markup firms does raise the aggregate markup.

Quantitatively, the decrease in software prices accounts for two-thirds of the labor share decline in Korea between 1990 and 2018. Slightly more than half of the effect comes through the markup channel (52 percent), and the rest (48 percent) through the factor substitution channel. One immediate implication is that the rise of the software or intangibles income share in the accounting sense, highlighted by Koh et al. (2020), will underestimate the role of software in the decline of the labor income share by more than 50 percent, as it misses the impact through the changes in markups. The effect through the markup channel can be attributed almost equally to within-firm markup growth and the between-firm reallocation. By contrast, nearly all the effect through the factor substitution channel is due to within-firm factor substitution.

By contrast, the decline in equipment prices has a negligible effect on the aggregate labor income share, because two opposing effects cancel each other out. The factor substitution channel pushes up the labor share because equipment and labor are complements within firms, even after factoring in the between-firm reallocation. The resulting rise in markups reduces the labor share.

The importance of markups and the between-firm reallocation for the labor share decline in Korea that we find is consistent with the findings of Autor et al. (2020) and Kehrig and Vincent (2021) for the US, although they do not specify the cause of the reallocation. The role of within-firm markup growth in our analysis aligns with the emphasis on within-firm sales growth in Kehrig and Vincent (2021).

In summary, it is software, not equipment, that substitutes for labor and reduces the labor income share. The resulting within-firm markup growth and the reallocation toward high markup firms are quantitatively important channels through which the fall in software prices reduces the aggregate labor share. Our analysis helps reconcile the macro vs. micro elasticity discord surrounding the labor share decline. It also shows that the rise of software is driving the well-documented reallocation toward large firms with high markups and low labor shares.

Related literature The elasticity of substitution between labor and capital is a crucial parameter in economic theory, and numerous studies have attempted to estimate it. One complication is that the elasticity at the level of individual firms or establishments may differ from the elasticity at the level of the aggregate economy. Oberfield and Raval (2021) provide a framework to compute the macro elasticity from micro elasticity estimates and the distribution of factor intensities across firms. Our work extends theirs to consider two distinct types of capital that can interact with labor in different ways and to allow endogenous changes in markups.

Related to our paper, Lashkari et al. (2023) study the interplay of capital-embodied technological change, labor share, and market concentration. Their focus is the role of information technology (IT), which has increasing returns to scale. Using French firm-level data, they find that the elasticity of substitution between IT capital (an amalgam of hardware and software) and labor is still less than one. (They assume that the elasticity of substitution between non-IT capital and labor is one.) Our paper separates software and equipment, and shows that software substitutes for labor but equipment does not. It also emphasizes the role of variable markups.[1]

Our work also relates to the macroeconomic literature on capital-embodied technological change, for example, Greenwood et al. (1997), Greenwood et al. (2000), and Cummins and Violante (2002). Krusell et al. (2000) showed that capital-embodied technological change and the complementarity between skilled labor and capital drove up the skill premium in the US, focusing exclusively on equipment capital. Our micro data does not allow us to differentiate labor by skill, but we did run a skilled vs. unskilled labor exercise using aggregate time series. We found that both skilled labor and unskilled labor are substitutes with respect to software, but skilled labor is less substitutable. Likewise, both types of labor are complements with respect to equipment capital, but skilled labor is more complementary. To the extent that the skill premium result of Krusell et al. (2000) only requires that skilled labor is more complementary or less substitutable with respect to capital than is unskilled labor, our findings are consistent.

This paper also relates to the research showing that the decline in the labor share

[1] A related paper is de Souza and Li (2023). It divides equipment into robots vs. tools, and finds evidence that robots substitute for labor but tools complement labor in Brazilian data.

partly reflects the misclassification of labor compensation as capital income, especially when workers are involved in the production or utilization of intangible capital. Koh et al. (2020) raised this possibility. Eisfeldt et al. (2023) document a rise in equity-based compensation of high-skill workers and show that their labor income share did not fall, when their equity-based compensation is accurately classified as labor income.

Some recent studies have pointed to the role of intangible capital in technological advancement (Corrado et al., 2009, 2022). However, few asked whether and how intangible capital and tangible capital may interact with other production factors in different ways. Exceptions include Aum and Shin (2020), which showed that industries with higher software intensity experienced a more rapid decline in labor share in the US. Another is Aum (2019), which documented that the correlation between labor and software across regions is distinct from the correlation between labor and equipment.[2] Our paper substantiates and rationalizes such suggestive evidence through a structural estimation of the micro and the macro elasticities of substitution between equipment and labor and between software and labor.

The data we use in our analysis predate the ebullience surrounding artificial intelligence (AI) and its implication for the economy unleashed by OpenAI's ChatGPT in 2022. Eisfeldt et al. (2023) provide some evidence that Generative AI, which is an extremely powerful class of software, is on net a substitute for labor, consistent with our finding regarding more conventional software.

The rest of the paper is structured as follows. We document relevant empirical facts in Section 2. In Section 3, we introduce our model framework. We then estimate micro elasticities and aggregate them into macro elasticities in Section 4. In Section 5, we quantify the contribution of software-embodied technological change to the decline in the aggregate labor income share through various channels. Section 6 concludes.

2 Motivating Facts

In this section, we describe the data used in our analysis and present the distinct empirical patterns exhibited by software and equipment.

Aggregate Trends Before going into the micro-level data, we provide an overview of the evolution of the labor income share and the capital income shares by capital

[2] Related, Park (2022) finds that an occupation's software intensity determines whether its employment share will grow or shrink during an investment boom.

type in Korea. The labor income share is labor income divided by the sum of labor income, capital income, and profits.[3] To calculate capital income, one needs estimates of the gross rate of return for each type of capital. We assume that the gross return R^j for capital type j satisfies the no-arbitrage condition:

$$R^j = (1+r)p^j_{t-1} - (1-\delta^j)p^j_t, \tag{1}$$

where r is the net rate of return, p^j_t is the price of capital j in period t, and δ^j is the depreciation rate of capital j. We use K^j, p^j, and δ^j from the National Accounts, and the corporate bond rate net of expected inflation to compute gross return on capital according to (1)—see Appendix B for details.

Fig. 1: Labor Income Share in Korea

Figure 1 plots the aggregate labor income share from 1980 to 2018, which declined over time, especially in the 1990s and the 2000s.

We plot the ratio between labor income and type-j capital income in log, $\log wL - \log R_j K_j$, in Figure 2. Software income grew faster than labor income (black solid line), whereas equipment income, if anything, decreased relative to labor income (gray solid line). The growth of software income relative to labor income was most pronounced

[3] Our measure of the labor income share is the gross labor share adjusted for proprietors' income (Gollin, 2002; Park, 2020). There have been debates about whether the decline in the net labor share in Korea is merely a measurement issue related to self-employment. Park (2020) showed that the net labor income share is sensitive to how one treats the changing share of "self-employed with employees," whose compensation is already included in total compensation. We follow Park (2020) and make adjustments only for "self-employed without employees." Our focus is on the gross labor share, because software, the production factor we are interested in, has a higher depreciation rate than other factors. The declining trend of the gross labor share is barely affected by the proprietor income adjustment. See Appendix B.1.3 for additional details.

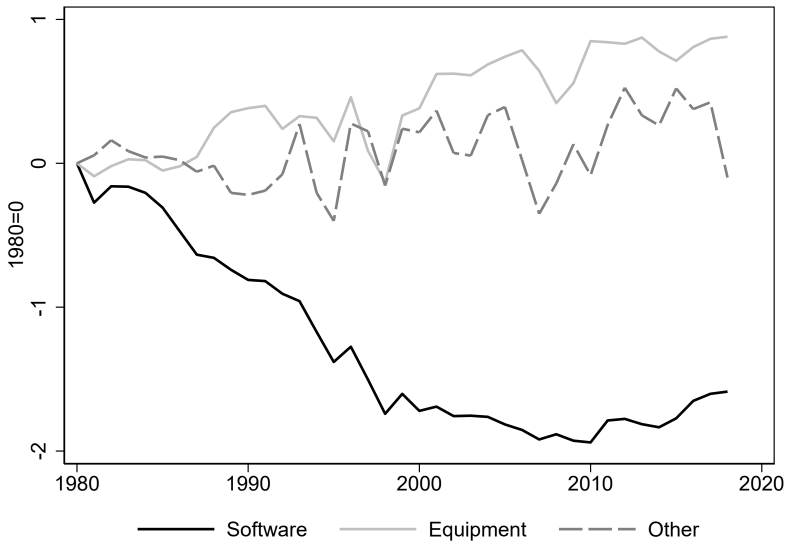


Fig. 2: Labor Income Relative to Capital Income by Capital Type

in the 1990s and the early 2000s, coinciding with the period of the steepest labor share decline in Figure 1.[4] This suggests that the fall in the aggregate labor income share has more to do with software than equipment.

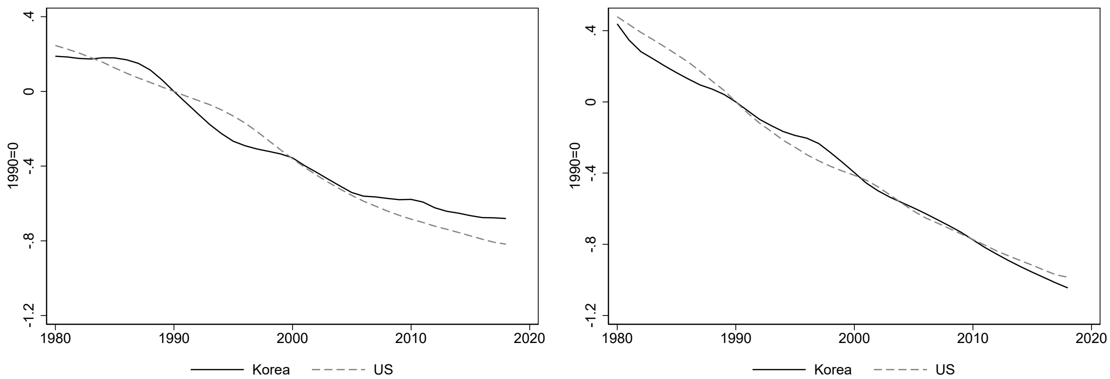
Researchers typically infer the technological change specific to capital goods from the declines in the relative price of investment to consumption (e.g., Cummins and Violante, 2002). Following the literature, we use the five-year moving average of the inverse of the price of software and equipment investment relative to consumption to measure the technological change embodied in them.[5]

Figure 3 plots the relative price of investment to consumption by capital type. Both equipment (left panel) and software (right panel) have experienced substantial capital-embodied technological change, inferred from the rapid decline in their relative price to consumption. It is noteworthy that software price fell more rapidly than equipment price.

These observations are not unique to Korea. For instance, the changes in the price

[4]The "other" category of capital (dashed line) in Figure 2 includes residential capital, structure, and R&D capital.

[5]One issue with the Korean National Accounts is that the software price index may underestimate software-embodied technological change. Since 1994, the price index of software investment comes from the producer price index, which may fail to capture quality improvements. To address this concern, we adjust the software price index following Parker and Grimm (2000), as does the US Bureau of Economic Analysis (BEA). BEA makes a bias adjustment of 3.15 percent per year to the producer price index from the Bureau of Labor Statistics to compensate for the discrepancy between the hedonic method and the matched model method.



(a) Equipment

(b) Software

Fig. 3: Relative Price of Investment to Consumption by Capital Type

The solid lines correspond to Korea, while the dashed lines correspond to the US. We compute five-year moving averages of the relative prices and normalize them to 0 in the year 1990.

of equipment and software relative to consumption are remarkably similar between Korea and the US, as shown by the solid and the dashed lines in both panels of Figure 3. In addition, consistent with the patterns in Figure 2, Aum and Shin (2020) show that an industry's software intensity is the one variable that is systematically correlated with its labor share decline, based on industry-level data from the US. However, the US micro-level data do not separate software from the broader category of intangibles, which comes with immense measurement problems. For this reason, we turn to micro-level data from Korea for our analysis.

Micro-level Data We use two datasets that have information on equipment and software as factors of production at the micro level. One is KISDATA, covering firms in Korea from 2003 to 2018. This dataset is a compilation of firm-level financial statements, provided by NICE Information Service. It covers firms listed in the Korea Stock Exchange, as well as those unlisted firms subject to external audit requirements.[6] Compared with the National Accounts, our KISDATA sample accounts for 47 and 56 percent of the compensation of employees and the operating surplus, respectively, of the entire non-financial corporate sector in 2018.

One key advantage of KISDATA is that it reports assets by type, for example, dis-

[6]The criteria for external audit requirement vary over time. Until 2008, firms whose asset exceeded 7 billion KRW (about 6.4 million USD in 2008) had to be audited externally. Since 2009, firms with (i) assets greater than 10 billion (12 billion since 2014) KRW, or (ii) assets greater than 7 billion KRW and liability greater than 7 billion KRW, or (iii) assets greater than 7 billion KRW and more than 300 employees had to undergo external audits.

tinguishing equipment from software. This is partly thanks to the Korea Generally Accepted Accounting Principles (K-GAAP). International Financial Reporting Standards (IFRS) are rule-based and mandate only a minimal number of items in financial statements. As a result, firm-level financial statements reported under IFRS usually do not disaggregate firms' assets by type. However, K-GAAP provides detailed financial statement accounts with standard formats.[7] This unique aspect of the Korean accounting principles enables us to investigate the separate roles of equipment and software. We construct firm-level labor compensation, value added, and software and equipment assets to construct labor share and capital intensities at the firm level. More details on data construction and summary statistics are in Appendix B.

The other dataset is the establishment-level data from the 2015 Korean Economic Census. Conducted every five years, the census surveys all establishments with at least one employee as of December 31 of that year. For those in the manufacturing sector, the census gathers data on intangible assets by type, including externally purchased software, for all uni-establishment firms and corporate headquarters of multi-establishment firms.[8] The census also contains information on annual payroll and equipment capital, among others. It also records the location of establishments, which enables us to utilize variations across regions.

Patterns in the Micro-level Data We begin by showing that firms' software intensity predicts a fall in labor share, but their equipment intensity makes no meaningful prediction. The regression equation is:

$$y_{i,t} = \gamma_i + \alpha_t + \beta_s s_{i,t-1} + \beta_e e_{i,t-1} + \varepsilon_{i,t} \,, \tag{2}$$

where $s_{i,t-1}$ is the software intensity (software asset divided by value added) of firm i in year $t-1$, $e_{i,t-1}$ is its equipment intensity (equipment asset divided by value added), γ_i is the firm fixed effect, and α_t is the time fixed effect.

The estimation results are in Table 1. In the KISDATA, firms' software intensity in year $t-1$ predicts a faster decline of their labor share (first column) and a higher sales growth (last column) between $t-1$ and t. Firms' equipment intensity, on the

[7]Since 2010, listed firms in Korea have adopted K-IFRS and no longer follow K-GAAP. However, companies that had followed K-GAAP before 2010 continue to report detailed accounts. In fact, the number of firms reporting software as a separate asset type has steadily increased since 2010.

[8]That is, the census surveys firm-level rather than establishment-level software assets, as intangible assets at the establishment level are not well defined. In our benchmark analysis, we assume each establishment of a multi-establishment firm uses the same amount of software as its headquarters—that is, software is non-rivalrous within a firm. In a robustness check, we limit our sample to uni-establishment firms and obtain nearly the same result.

	ΔLabor Share			Δ log Sales
	Total	Non-production	Production	
$s_{i,t-1}$	-0.287***	-0.195***	-0.102***	0.392***
	(0.049)	(0.044)	(0.013)	(0.126)
$e_{i,t-1}$	-0.009	-0.006	-0.004	0.030**
	(0.007)	(0.004)	(0.003)	(0.015)
N	44,364	43,759	40,626	44,356
R^2	0.189	0.207	0.191	0.291

Table 1: Capital Intensities, Labor Share and Sales Change at the Firm Level

Standard errors are clustered at the firm level. *, **, and *** indicate significance at the 10, 5, and 1 percent level, respectively.

other hand, is not significantly related to changes in the labor share, although it does predict faster sales growth. These results suggest that software may be the driving force behind the empirical fact documented by Kehrig and Vincent (2021): those establishments whose labor share fell and sales increased at the same time account for most of the aggregate labor share decline in the US manufacturing sector.

Another finding is that software intensity predicts a steeper fall in the income share of non-production workers than that of production workers (second and third columns). This finding suggests that different workers may be affected differently by the rise of software, but the limited dimension of worker heterogeneity in our micro-level data (essentially, production vs. non-production workers) discourages a richer analysis in this direction.

Aggregating the manufacturing establishments in the census to the region level (Si-Gun-Gu, of which there are 162), we compute the correlation between the average wage of a region and the shares of capital income by capital type in the region.[9] Figure 4 depicts the relationship between the log of the ratio of regional expenditures on capital (either software or equipment) to labor on the y-axis against the log of local wages on the x-axis. The ratio of software to labor expenditures tends to be higher in regions with higher local wages (solid gray line). The ratio of equipment to labor expenditures shows, if anything, a negative correlation with local wages (black dashed line). This cross-sectional pattern complements the time-series evidence in Figure 2, suggesting that the decline in the aggregate labor share has more to do with software than equipment. More specifically, the finding that regions with higher wages spend

[9] Regional average wages are constructed from yet another dataset, Regional Employment Survey, controlling for various worker characteristics.

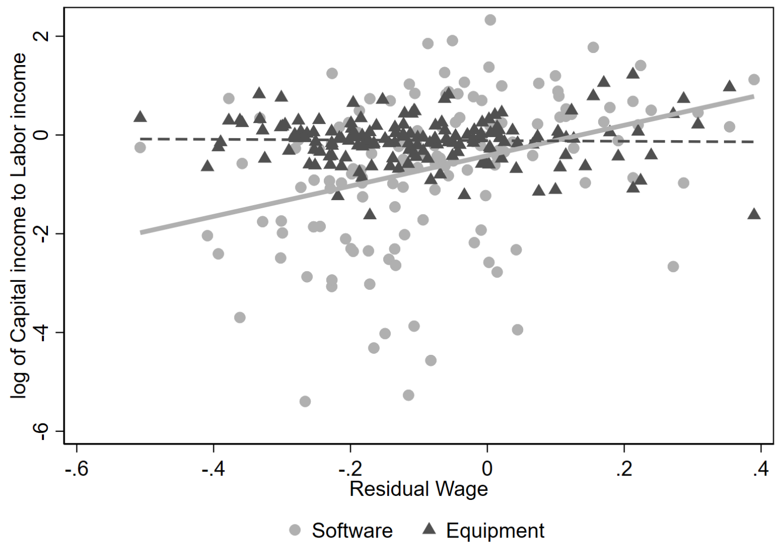


Fig. 4: Relationship between Local Wage and Factor Shares

Each dot represents a region (Si-Gun-Gu, an administrative unit in Korea). A region's average wage is from the Regional Employment Survey, controlled for workers' education, age, gender, and experience.

relatively more on software but not on equipment points to the possibility that software substitutes for labor more than does equipment.

To summarize, the patterns in the Korean data suggest that software may be the key to understanding the decline in the aggregate labor income share. At the aggregate level, it is software income, not equipment income, that grew at a faster rate than labor income. At the firm level, software intensity predicts a faster decline in labor income share, but equipment intensity does not. Furthermore, in regions with higher wages, establishments have higher ratios of software income to labor income, but no such relationship exists for the ratio between equipment income and labor income. Motivated by the multitudes of suggestive evidence, we now turn to a more rigorous and definitive analysis of the role of software in the decline of labor share and the rise of markups, both within firms and in the aggregate through between-firm reallocation.

3 Theory

3.1 Model

To investigate the role of software in shaping factor income shares, we consider a production function that has software and equipment as separate inputs, as well as labor and materials. We define a production of firm i as follows.

$$Y_i = \left\{ \left[\left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e(\sigma_S - 1)}{(\sigma_e - 1)\sigma_S}} + \alpha_i^S (A_S S_i)^{\frac{\sigma_S - 1}{\sigma_S}} \right]^{\frac{\sigma_S(\sigma_m - 1)}{(\sigma_S - 1)\sigma_m}} + \alpha_i^M M_i^{\frac{\sigma_m - 1}{\sigma_m}} \right\}^{\frac{\sigma_m}{\sigma_m - 1}} \tag{3}$$

Y_i is output, L_i is labor, K_i is tangible capital (or equipment), S_i is software, and M_i is material input. α_i^L, α_i^K, α_i^S, and α_i^M are the intensities of labor, equipment, software, and material input, respectively. All the inputs and intensities have subscript i, the firm index. A_L, A_K, and A_S represent economy-wide factor-augmenting technologies for labor, equipment, and software, respectively. For notational convenience, we also define the value added V_i and the equipment-labor bundle X_i as follows.

$$V_i \equiv \left[\left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e(\sigma_S - 1)}{(\sigma_e - 1)\sigma_S}} + \alpha_i^S (A_S S_i)^{\frac{\sigma_S - 1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S - 1}} \tag{4}$$

$$X_i \equiv \left(\alpha_i^L (A_L L_i)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_i^K (A_K K_i)^{\frac{\sigma_e - 1}{\sigma_e}} \right)^{\frac{\sigma_e}{\sigma_e - 1}}. \tag{5}$$

We note two properties of the production function in equation (3) that are relevant for our analysis. First, two parameters separately govern the elasticity of substitution between labor and equipment (σ_e) and the elasticity between labor and software (σ_s). That is, we can capture different labor share responses to technological changes embodied in different types of capital. Second, all firms are different in how intensively they use each factor, as captured by α_i's, implying that they will be affected differently by an economy-wide factor-augmenting technological change. As a result, aggregate changes in factor income shares depend not only on within-firm adjustments but also on the reallocation of the production activity across firms. In other words, the elasticity of substitution at the aggregate level will be different from the elasticity of substitution at the firm level, as Oberfield and Raval (2021) showed using a two-factor production function.

To allow for the possibility that software may trigger changes in markup within firms and in the aggregate, we consider a Kimball (1995) aggregator. Specifically, we assume that aggregate production Y satisfies

$$\sum_i H(Y_i/Y) = 1, \tag{6}$$

where $H(\cdot)$ is an increasing and concave function and Y_i is the firm-level production in equation (3). This aggregator implies the demand for Y_i given by

$$\frac{p_i}{p} = H'\left(\frac{Y_i}{Y}\right) \text{ or } \frac{Y_i}{Y} = h\left(\frac{p_i}{p}\right), \tag{7}$$

where h is the inverse function of H'. When $H(x) = x^{\frac{\epsilon-1}{\epsilon}}$, we obtain a standard CES demand system with the constant elasticity of substitution ϵ.

From equation (7), $d\ln(Y_i/Y) = [(h'(p_i/p)p_i/p)/h(p_i/p)] \times d\ln(p_i/p)$, which gives the demand elasticity

$$\epsilon_i = -\frac{h'(p_i/p)p_i/p}{h(p_i/p)}.$$

Firm i takes the factor prices of labor (w), equipment (r), software (q), and the price of intermediate input (v) as given. A firm's profit maximization is

$$\max_{p_i, Y_i, K_i, S_i, M_i} p_i Y_i - wL_i - rK_i - qS_i - vM_i,$$

subject to equations (3) and (7). The first-order conditions are:

$$w = p_i \alpha_i^L A_L^{\frac{\sigma_e-1}{\sigma_e}} \left(\frac{Y_i}{V_i}\right)^{\frac{1}{\sigma_m}} \left(\frac{V_i}{X_i}\right)^{\frac{1}{\sigma_s}} \left(\frac{X_i}{L_i}\right)^{\frac{1}{\sigma_e}}, \tag{8}$$

$$r = p_i \alpha_i^K A_K^{\frac{\sigma_e-1}{\sigma_e}} \left(\frac{Y_i}{V_i}\right)^{\frac{1}{\sigma_m}} \left(\frac{V_i}{X_i}\right)^{\frac{1}{\sigma_s}} \left(\frac{X_i}{K_i}\right)^{\frac{1}{\sigma_e}}, \tag{9}$$

$$q = p_i \alpha_i^S A_S^{\frac{\sigma_s-1}{\sigma_s}} \left(\frac{Y_i}{V_i}\right)^{\frac{1}{\sigma_m}} \left(\frac{V_i}{S_i}\right)^{\frac{1}{\sigma_s}}, \tag{10}$$

$$v = p_i \alpha_i^M \left(\frac{Y_i}{M_i}\right)^{\frac{1}{\sigma_m}}, \tag{11}$$

$$p_i = \mu_i(p_i/p) \left\{ \left[\phi_i^{1-\sigma_s} + \alpha_i^{S^{\sigma_s}} \left(\frac{q}{A_S}\right)^{1-\sigma_s} \right]^{\frac{1-\sigma_m}{1-\sigma_s}} + \alpha_i^{M^{\sigma_m}} v^{1-\sigma_M} \right\}^{\frac{1}{1-\sigma_M}}, \tag{12}$$

where $\mu_i(p_i/p)$ is the optimal markup that satisfies $\mu_i(p_i/p) = \epsilon_i(p_i/p)/(\epsilon_i(p_i/p) - 1)$. For notational convenience, we introduced ϕ_i, the price of the equipment-labor bundle X_i:

$$\phi_i \equiv \left(\alpha_i^{L^{\sigma}}{}_e \left(\frac{w}{A_L}\right)^{1-\sigma_e} + \alpha_i^{K^{\sigma}}{}_e \left(\frac{r}{A_K}\right)^{1-\sigma_e} \right)^{\frac{1}{1-\sigma_e}}. \tag{13}$$

The factor market clearing conditions are

$$L = \sum_i L_i,\ K = \sum_i K_i,\ \text{and } S = \sum_i S_i. \tag{14}$$

Given L, K, and S, we can solve for the equilibrium from equations (3), (6), (7), (8), (9), (10), (12), and (14).

3.2 Elasticity of Substitution between Factors

3.2.1 Micro-level Elasticity

We now analyze the changes in factor shares in response to factor-augmenting technological changes or exogenous changes in factor prices, which are governed by the elasticity of substitution.

When there are more than two factors of production, there are multiple ways to define the elasticity of substitution between any pair of factors. For example, the elasticity of substitution between labor and equipment will vary, depending on whether we fix both output and software or fix only output and allow software to adjust (Stern, 2011). Therefore, we need to first clarify what we call the elasticity of substitution between factors.

We focus on the simplest one, the Allen-Uzawa elasticity. The Allen-Uzawa elasticity of substitution between factors x and y is defined as

$$\sigma_{x,y} = \frac{CC_{xy}}{C_x C_y},$$

where C is the cost function and C_x is its partial derivative with respect to the change in the price of input x, while all other prices are held constant. In our framework, the Allen-Uzawa elasticity of substitution between labor and equipment is simply σ_e, and the one between labor and software is σ_s in the production function in equation (3).

Proposition 1 (Micro-level elasticity of substitution) *The production function parameters σ_e and σ_s satisfy the following.*

$$\sigma_e = 1 + \frac{d\ln rK_i/wL_i}{d\ln w/r} = 1 + \frac{d\ln k_i/(1-k_i)}{d\ln w/r}, \tag{15}$$

$$\sigma_s = 1 + \frac{d\ln qS_i/(wL_i + rK_i)}{(1-k_i)dlnw/q + k_i d\ln r/q} = 1 + \frac{d\ln s_i/(1-s_i)}{d\ln \phi_i/q}. \tag{16}$$

Proof *In Appendix A.* ∎

For convenience, we introduce new notations for the factor shares of costs within firm i:

$$k_i \equiv \frac{rK_i}{wL_i + rK_i},\ \ell_i \equiv \frac{wL_i}{wL_i + rK_i + qS_i},$$

$$\kappa_i \equiv \frac{rK_i}{wL_i + rK_i + qS_i},\; s_i \equiv \frac{qS_i}{wL_i + rK_i + qS_i},\; m_i \equiv \frac{vM_i}{wL_i + rK_i + qS_i + vM_i}.$$

We denote firm i's equipment share of expenditures on equipment-labor bundle as $k_i \equiv rK_i/(wL_i + rK_i)$. Similarly, $\ell_i \equiv wL_i/(wL_i + rK_i + qS_i)$ is firm i's labor share of non-material cost, $\kappa_i \equiv rK_i/(wL_i + rK_i + qS_i)$ is firm i's equipment share of non-material cost, and $s_i \equiv qS_i/(wL_i + rK_i + qS_i)$ is firm i's software share of non-material cost. Lastly, $m_i \equiv \nu M_i/(wL_i + rK_i + qS_i + \nu M_i)$ refers to firm i's materials share of its total cost.

Proposition 1 establishes that the direction of change in factor shares in response to input price changes depends on whether σ_e and σ_s are greater than one or not. For example, when the price of equipment (r) falls, the ratio of labor expenditure to equipment expenditure (wL_i/rK_i) falls when $\sigma_e > 1$ but rises when $\sigma_e < 1$. Similarly, when the price of software (q) falls, software expenditure relative to expenditures on labor and equipment ($qS_i/(wL_i + rK_i)$) rises when $\sigma_s > 1$ but falls when $\sigma_s < 1$.

We obtain the following corollary that input price changes and factor-augmenting technological changes have equivalent effects on factor shares.

Corollary 1 *The production function parameters σ_e and σ_s satisfy the following.*

$$\sigma_e = 1 + \frac{d\ln rK_i/wL_i}{d\ln A_K/A_L} = 1 + \frac{d\ln k_i/(1-k_i)}{d\ln A_K/A_L}, \tag{17}$$

$$\sigma_s = 1 + \frac{d\ln qS_i/(wL_i + rK_i)}{(1-k_i)dlnA_S/A_L + k_i d\ln A_S/A_K} = 1 + \frac{d\ln s_i/(1-s_i)}{d\ln \phi_i A_S}. \tag{18}$$

Proof *In Appendix A.* ∎

In other words, we cannot separately identify the impacts of factor-augmenting technological changes from the impacts of factor price changes on factor shares. One implication is that it is difficult to estimate the elasticities of substitution using aggregate time-series data on factor shares and input prices, since factor-augmenting technological changes are typically unobservable.

3.2.2 Macro-level Elasticity

Since we do not restrict how factor shares or equivalently α_i's in equation (3) are distributed across firms, the model does not give a well-defined aggregate production function. However, as in Oberfield and Raval (2021), we can still derive the relationship between the firm-level elasticities of substitution in Proposition 1 and the changes in the aggregate factor income shares in response to factor price changes. The latter is the aggregate elasticity of substitution, defined as follows. Note that going from the usual two factors of production to three requires a new approach.

Definition 1 (Aggregate elasticity of substitution) *The aggregate elasticities of substitution $\bar{\sigma}_e^w$, $\bar{\sigma}_e^r$, $\bar{\sigma}_s^w$, $\bar{\sigma}_s^r$, and $\bar{\sigma}_s^q$ are:*

$$\bar{\sigma}_e^w \equiv 1 + \frac{d \ln rK/wL}{d \ln w}, \tag{19}$$

$$\bar{\sigma}_e^r \equiv 1 - \frac{d \ln rK/wL}{d \ln r}, \tag{20}$$

$$\bar{\sigma}_s^w \equiv 1 + \frac{d \ln qS/(wL + rK)}{wL/(wL + rK) \times d \ln w}, \tag{21}$$

$$\bar{\sigma}_s^r \equiv 1 + \frac{d \ln qS/(wL + rK)}{rK/(wL + rK) \times d \ln r}, \tag{22}$$

$$\bar{\sigma}_s^q \equiv 1 - \frac{d \ln qS/(wL + rK)}{d \ln q}, \tag{23}$$

Unlike in Oberfield and Raval (2021), we need to define the aggregate elasticity of substitution between factors with respect to each input price w, r, and q. That is, even for the same change in the relative factor price, w/r for example, the corresponding change in the aggregate factor shares will depend on whether the wage increased or the equipment price fell. This is because we have three factors of production (equipment, software, and labor), not just two. For example, a firm with a low labor cost share ℓ_i may also have a low equipment cost share κ_i, if its software cost share s_i is high. In this case, firms that lose less than average from a wage increase do not necessarily benefit more from a fall in equipment price. Accordingly, the reallocation across firms will depend on which factor price changes, and the change in the aggregate labor share in response to a higher wage can be different from the change in response to a lower equipment price. This is not the case when there are only two factors, because firms' labor share is equal to one minus their equipment share—that is, a perfect negative correlation between factor shares.[10]

In addition, because our model features variable markups, each firm faces a different demand elasticity. When the price of equipment falls, for example, it makes equipment-intensive firms effectively more productive than others. If more equipment-intensive firms face a lower demand elasticity (or higher markup), there is less reallocation than in the case with a constant markup. With variable markups,

[10] The presence of material inputs in the production function (3) does not necessarily imply asymmetry in the elasticity of substitution. This is because material inputs are a combination of factor inputs such as labor, capital, and software, and hence any change in material price is endogenously determined by the intensity of each factor input in the material goods production. We assume that factor intensities in the material goods production are the same as those in the final good production. Related, Oberfield and Raval (2021) has materials as another factor in the production function, but they obtain symmetry between labor and capital by assuming that changes in the relative price of materials to capital are proportional to changes in the relative price of labor to capital. Essentially, materials are produced by combining labor and capital (in a Cobb-Douglas manner), so there are only two factors of production.

the magnitude of reallocation across firms depends on how factor intensities are correlated with firms' markups.[11] Again, with three inputs, the fact that labor shares are positively correlated with markups does not necessarily mean that equipment shares are negatively correlated with markups.

Now we derive our main proposition that links firm-level elasticities of substitution to the aggregate elasticities of substitution. We introduce additional notations for convenience:

$$
\begin{aligned}
&k \equiv \frac{rK}{wL+rK},\ \ell \equiv \frac{wL}{wL+rK+qS},\ \kappa \equiv \frac{rK}{wL+rK+qS},\ s \equiv \frac{qS}{wL+rK+qS},\\
&m \equiv \frac{vM}{wL+rK+qS+vM},\ \theta_i \equiv \frac{wL_i+rK_i}{wL+rK},\ \omega_i \equiv \frac{wL_i+rK_i+qS_i}{wL+rK+qS},\\
&\gamma_i \equiv \frac{wL_i+rK_i+qS_i+vM_i}{wL+rK+qS+vM},
\end{aligned}
$$

where k denotes aggregate equipment share of aggregate expenditures on labor and equipment, ℓ is the aggregate labor expenditure share of aggregate non-material cost, κ is the aggregate equipment share of aggregate non-material cost, s_i is the aggregate software share of aggregate non-material cost, and m is the aggregate materials share of aggregate total cost. In addituion, θ_i denotes firm i's expenditures on labor and equipment as a fraction of aggregate expenditures on labor and equipment, ω_i is firm i's share of aggregate expenditures on non-material inputs, and γ_i is firm i's share of the aggregate total expenditure.

Proposition 2 (Aggregation) *The aggregate elasticities of substitution satisfy*

$$\bar{\sigma}_e^w = (1-\chi)\sigma_e + \chi\left[\zeta^w\sigma_s + (1-\zeta^w)\bar{m}_e^w\sigma_m + (1-\zeta^w-(1-\zeta^w)\bar{m}_e^w)\,\bar{\epsilon}_e^w\right], \tag{24}$$

$$\bar{\sigma}_e^r = (1-\chi)\sigma_e + \chi\left[\zeta^r\sigma_s + (1-\zeta^r)\bar{m}_e^r\sigma_m + (1-\zeta^w-(1-\zeta^r)\bar{m}_e^r)\,\bar{\epsilon}_e^r\right], \tag{25}$$

$$\bar{\sigma}_s^q = (1-\xi^q)\sigma_s + \xi^q\left[\bar{m}_s^q\sigma_m + (1-\bar{m}_s^q)\bar{\epsilon}_s^q\right], \tag{26}$$

$$\bar{\sigma}_s^w = (1-\xi^w)\sigma_s + \xi^w\left[\bar{m}_s^w\sigma_m + (1-\bar{m}_s^w)\bar{\epsilon}_s^w\right], \tag{27}$$

$$\bar{\sigma}_s^r = (1-\xi^r)\sigma_s + \xi^r\left[\bar{m}_s^r\sigma_m + (1-\bar{m}_s^r)\bar{\epsilon}_s^r\right], \tag{28}$$

where

$$
\begin{aligned}
&\chi \equiv \frac{\sum_i (k_i-k)^2\theta_i}{k(1-k)},\ \zeta^w \equiv \frac{\sum_i (k_i-k)(1-k_i)\theta_i s_i}{\sum_i (k_i-k)(1-k_i)\theta_i},\ \zeta^r \equiv \frac{\sum_i (k_i-k)k_i\theta_i s_i}{\sum_i (k_i-k)k_i\theta_i},\\
&\bar{m}_e^w \equiv \frac{\sum_i (k_i-k)(\ell_i-\alpha_p^w)\theta_i m_i}{\sum_i (k_i-k)(\ell_i-\alpha_p^w)\theta_i},\ \bar{m}_e^r \equiv \frac{\sum_i (k_i-k)(\kappa_i-\alpha_p^r)\theta_i m_i}{\sum_i (k_i-k)(\kappa_i-\alpha_p^r)\theta_i},\\
&\xi^q \equiv \frac{\sum_i \omega_i(s_i-s)^2}{s(1-s)},\ \xi^w \equiv -\frac{\sum_i \omega_i(s_i-s)(\ell_i-\ell)}{s\ell},\ \xi^r \equiv -\frac{\sum_i \omega_i(s_i-s)(\kappa_i-\kappa)}{s\kappa},
\end{aligned}
$$

[11] Another implication from variable markup is that relative changes in factor share of cost are not equal to relative changes in factor share of income. We discuss this issue in Section 3.3.

$$\bar{\epsilon}_e^w \equiv \frac{\sum_i((1-k_i)-(1-k))[\ell_i(1-m_i)-\alpha_p^w]\theta_i\epsilon_i b_i}{\sum_i((1-k_i)-(1-k))[\ell_i(1-m_i)-\alpha_p^w]\theta_i},\ \bar{\epsilon}_e^r \equiv \frac{\sum_i(k_i-k)(\kappa_i(1-m_i)-\alpha_p^r)\theta_i\epsilon_i b_i}{\sum_i(k_i-k)(\kappa_i(1-m_i)-\alpha_p^r)\theta_i}$$

$$\bar{m}_s^q \equiv \frac{\sum_i(s_i-s)(s_i-\alpha_p^q)\omega_i m_i}{\sum_i(s_i-s)(s_i-\alpha_p^q)\omega_i},\ \bar{m}_s^w \equiv \frac{\sum_i(s_i-s)(\ell_i-\alpha_p^w)\omega_i m_i}{\sum_i(s_i-s)(\ell_i-\alpha_p^w)\omega_i},\ \bar{m}_s^r \equiv \frac{\sum_i(s_i-s)(\kappa_i-\alpha_p^r)\omega_i m_i}{\sum_i(s_i-s)(\kappa_i-\alpha_p^r)\omega_i}$$

$$\bar{\epsilon}_s^q \equiv \frac{\sum_i(s_i-s)(1-m_i)(s_i-\alpha_p^q)\omega_i\epsilon_i b_i}{\sum_i(s_i-s)(1-m_i)(s_i-\alpha_p^q)\omega_i},\ \bar{\epsilon}_s^w \equiv \frac{\sum_i(s_i-s)(1-m_i)(\ell_i-\alpha_p^w)\omega_i\epsilon_i b_i}{\sum_i(s_i-s)(1-m_i)(\ell_i-\alpha_p^w)\omega_i},$$

$$\bar{\epsilon}_s^r \equiv \frac{\sum_i(s_i-s)(1-m_i)(\kappa_i-\alpha_p^r)\omega_i\epsilon_i b_i}{\sum_i(s_i-s)(1-m_i)(\kappa_i-\alpha_p^r)\omega_i},\ b_i(x) \equiv \frac{1}{1-\mu'(x)x/\mu(x)}\left(=\frac{d\ln p_i}{d\ln mc_i}\right)$$

$$\alpha_p^w \equiv \frac{\sum_i p_i Y_i(1-\epsilon_i)b_i(1-m_i)\ell_i}{\sum_i p_i Y_i(1-\epsilon_i)b_i(1-m_i)},\ \alpha_p^r \equiv \frac{\sum_i p_i Y_i(1-\epsilon_i)b_i(1-m_i)\kappa_i}{\sum_i p_i Y_i(1-\epsilon_i)b_i(1-m_i)},$$

$$\alpha_p^q \equiv \frac{\sum_i p_i Y_i(1-\epsilon_i)b_i(1-m_i)s_i}{\sum_i p_i Y_i(1-\epsilon_i)b_i(1-m_i)}$$

Proof *In Appendix A.* ∎

Proposition 2 is our main theoretical result. The aggregate elasticity of substitution is a weighted average of the micro elasticity of substitution and those cross-sectional moments that govern between-firm reallocation. The weights themselves are moments of the joint distribution of factor shares and markups across firms.

The aggregate elasticities of substitution between labor and equipment, $\bar{\sigma}_e^w$ and $\bar{\sigma}_e^r$ (with respect to wage and equipment price, respectively), for example, are weighted averages of the micro elasticities σ_e, σ_s, σ_m, and $\bar{\epsilon}$, with $\bar{\epsilon}$ being the weighted average of the demand elasticity across firms. The weight χ in equations (24) and (25) is proportional to the variance of equipment share of labor and equipment expenditures k_i across firms. Intuitively, when k_i's are more dispersed, reallocation across firms becomes more important, putting more weights on the demand elasticity that governs the reallocation. If all firms had the same equipment intensity k_i, they will adjust the factor expenditure ratios by the same proportion, and the aggregate elasticity would be the same as the micro or within-firm elasticity.[12] In these equations, $\bar{\epsilon}_e^w$ and $\bar{\epsilon}_e^r$ are weighted averages of $\epsilon_i b_i$ across firms, where b_i is the responsiveness of firm i's price to a change in its marginal cost ($b_i \equiv d\ln p_i/d\ln mc_i$)—a measure of pass-through. The reallocation depends on how elastic a firm's demand is (ϵ_i) and how much markup changes when the marginal cost changes (b_i).

We now discuss the aggregate elasticities of substitution between software and the labor-equipment bundle, shown in equations (26)–(28). Again, the change in the

[12] In equations (24) and (25), the between-firm reallocation depends on σ_s as well as $\bar{\epsilon}$. For these elasticities between labor and equipment, what matters for reallocation is firm i's share of the aggregate labor-equipment bundle, which can be obtained from its share of the aggregate non-material cost and the distribution of software share of non-material cost (i.e., $\theta_i = \omega_i(1-s_i)/(1-s)$). This is why σ_s appears.

software share of non-material cost depends on which factor price changes (software price q, wage w, or equipment price r).

Suppose that the price of equipment goes down. Then the price of the equipment-labor composite goes down, causing firms to substitute away from software if the micro elasticity σ_s is greater than one. The magnitude of the decline in the bundle price depends on how intensively a firm uses equipment, with the within-firm substitution being more prominent in firms with high κ_i (equipment share of non-material cost). It follows that the substitution away from software in the aggregate in response to a lower equipment price is larger when high κ_i firms, who become larger, tend to have a lower s_i (software share of non-material cost). This is why ξ^r is proportional to the minus of the covariance between κ_i and s_i. Again, a large κ_i does not necessarily imply a small s_i because we have three inputs and we do not restrict the factor share distribution. If the covariance between κ_i and s_i were positive, the reallocation responding to a lower equipment price will counteract the substitution away from software within firms.

Following the same logic, ξ^w in equation (27) is proportional to the minus of the covariance between ℓ_i (labor share of non-material cost) and s_i (software share of non-material cost).

Lastly, the change in software share in response to a change in software price in equation (26) depends more on between-firm reallocation when s_i's are more dispersed (ξ^q is proportional to the variance of s_i). Given the dispersion ξ_q, the reallocation is smaller (i.e., lower $\bar{\epsilon}_s^q$) when a firm with higher software share (s_i) faces a less elastic demand (smaller ϵ_i or higher μ_i) or a less responsive markup (lower b_i).

3.3 Changes in the Aggregate Markup

When factor prices change, a firm adjusts its output price accordingly. Under the Kimball aggregator in equation (6), the magnitude of output price adjustment differs across firms, and as a result the aggregate markup changes. The following proposition summarizes how the aggregate markup changes in response to changes in factor prices.

Proposition 3 (Aggregate markup) *The change in aggregate markup in response to factor price changes is respectively given by*

$$\bar{\sigma}_\mu^w - 1 \equiv \frac{d \ln \mu}{\tilde{\ell} d \ln w} = (1 - \eta^w)(\bar{b}^w - 1) + \eta^w(\bar{\epsilon}_\mu^w - 1) + (\iota^w - \eta^w)(\sigma_m - 1) \,, \tag{29}$$

$$\bar{\sigma}_\mu^r - 1 \equiv \frac{d \ln \mu}{\tilde{\kappa} d \ln r} = (1 - \eta^r)(\bar{b}^r - 1) + \eta^r(\bar{\epsilon}_\mu^r - 1) + (\iota^r - \eta^r)(\sigma_m - 1) \,, \tag{30}$$

$$\bar{\sigma}_\mu^q - 1 \equiv \frac{d\ln\mu}{\tilde{s}d\ln q} = (1-\eta^q)(\bar{b}^q - 1) + \eta^q(\bar{\epsilon}_\mu^q - 1) + (\iota^q - \eta^q)(\sigma_m - 1)\,, \tag{31}$$

where

$$\begin{aligned}
&\mu \equiv \sum_i \omega_i\mu_i,\ \tilde{\ell} \equiv \sum_i \omega_i\tilde{\ell}_i,\ \tilde{\kappa} \equiv \sum_i \omega_i\tilde{\kappa}_i,\ \tilde{s} \equiv \sum_i \omega_i\tilde{s}_i,\\
&\tilde{\ell}_i \equiv (1-m_i)\ell_i + m_i\alpha_p^w,\ \tilde{\kappa}_i \equiv (1-m_i)\kappa_i + m_i\alpha_p^r,\ \tilde{s}_i \equiv (1-m_i)s_i + m_i\alpha_p^q,\\
&\bar{b}^w \equiv \frac{\sum_i \mu_i\tilde{\ell}_i\omega_i b_i}{\sum_i \mu_i\tilde{\ell}_i\omega_i},\ \bar{b}^r \equiv \frac{\sum_i \mu_i\tilde{\kappa}_i\omega_i b_i}{\sum_i \mu_i\tilde{\kappa}_i\omega_i},\ \bar{b}^q \equiv \frac{\sum_i \mu_i\tilde{s}_i\omega_i b_i}{\sum_i \mu_i\tilde{s}_i\omega_i},\\
&\eta^w \equiv -\frac{\sum_i(\mu_i-\mu)(\tilde{\ell}_i-\tilde{\ell})\omega_i}{\tilde{\ell}\mu},\ \iota^w \equiv -\frac{\sum_i(\mu_i-\mu)(\ell_i-\ell)\omega_i}{\tilde{\ell}\mu},\\
&\bar{\epsilon}_\mu^w \equiv \frac{\sum_i(\mu_i-\mu)(\tilde{\ell}_i-\alpha_p^w)\omega_i\epsilon_i b_i}{\sum_i(\mu_i-\mu)(\tilde{\ell}_i-\alpha_p^w)\omega_i}\\
&\eta^r \equiv -\frac{\sum_i(\mu_i-\mu)(\tilde{\kappa}_i-\tilde{\kappa})\omega_i}{\tilde{\kappa}\mu},\ \iota^r \equiv -\frac{\sum_i(\mu_i-\mu)(\kappa_i-\kappa)\omega_i}{\tilde{\kappa}\mu},\\
&\bar{\epsilon}_\mu^r \equiv \frac{\sum_i(\mu_i-\mu)(\tilde{\kappa}_i-\alpha_p^r)\omega_i\epsilon_i b_i}{\sum_i(\mu_i-\mu)(\tilde{\kappa}_i-\alpha_p^r)\omega_i}\\
&\eta^q \equiv -\frac{\sum_i(\mu_i-\mu)(\tilde{s}_i-\tilde{s})\omega_i}{\tilde{s}\mu},\ \iota^q \equiv -\frac{\sum_i(\mu_i-\mu)(s_i-s)\omega_i}{\tilde{s}\mu},\\
&\bar{\epsilon}_\mu^q \equiv \frac{\sum_i(\mu_i-\mu)(\tilde{s}_i-\alpha_p^q)\omega_i\epsilon_i b_i}{\sum_i(\mu_i-\mu)(\tilde{s}_i-\alpha_p^q)\omega_i}
\end{aligned}$$

Proof *In Appendix A.* ∎

Similar to the aggregate elasticities of substitution between production factors, a change in aggregate markup can be represented by a weighted sum of within-firm changes and between-firm changes. That is, a change in aggregate markup depends not only on how each firm adjusts its markup, but also on the reallocation of production activity across firms. Consider a fall in software price ($d\ln q < 0$) as an example. For each firm, a fall in software price reduces its marginal cost according to $d\ln mc_i/d\ln q = s_i(1-m_i) + m_i\alpha_p^q$, where $s_i(1-m_i)$ is the share of software expenditure relative to total cost and α_p^q is a share of software contents in materials (Shephard's lemma). As the responsiveness of a firm's price to its marginal cost (or pass-through) is denoted by $b_i \equiv d\ln p_i/d\ln mc_i$, the within-firm markup change is $-(b_i-1)[s_i(1-m_i)+m_i\alpha_p^q]$.[13] The first term in the right-hand side of equation (31), $\bar{b}^q - 1$, captures a weighted average of these within-firm adjustments across firms.

Now consider the reallocation across firms, or the between-firm change. When software price falls ($d\ln q < 0$), the reallocation makes aggregate markup increase if

[13] For example, if $b_i < 1$ as in Klenow and Willis (2016), firms increase their markup in response to the fall in software price.

$\eta^q < 0$ in equation (31). That is, the reallocation raises the aggregate markup when software shares and markups are positively correlated in the data (see the definition of η^q in the second-to-last line of Proposition 3), because software-intensive firms' weights rise with a fall in software price.

The degree of reallocation $\bar{\epsilon}_\mu^q$ in equation (31) is determined by the weighted average of the demand elasticity ϵ_i and the pass-through b_i across firms—see the last equation of Proposition 3. The product $\epsilon_i b_i$ determines how firm i's market share changes in response to the change in software price. The weights are larger for software-intensive firms, since the marginal cost change is proportional to firms' software share s_i. Intuitively, when firms with higher software shares adjust their size more, there is more reallocation across firms in response to a fall in software price.

Note that the weight is firm i's share of the aggregate value added (ω_i) rather than its share of the total cost (γ_i) when aggregating markup. A change in total cost share is not exactly proportional to a change in value added share, except when $\sigma_m = 1$. As a result, the adjustment term $(\iota^q - \eta^q)(\sigma_m - 1)$ appears at the end of the equation (31).

How factor shares correlate with markup is an empirical question. When a factor share is positively correlated with markup, its covariance is positive and the corresponding η is negative. The degrees of reallocation $\bar{\epsilon}_\mu$'s themselves can reinforce or counteract the within-firm markup changes depending on the joint distribution of markup and factor shares in cost.

The aggregate labor income share is the aggregate labor cost share in aggregate non-material cost divided by the aggregate markup, ℓ/μ. To see this, let firm i's non-material cost $C_i \equiv wL_i + rK_i + qS_i$ and aggregate non-material cost $C \equiv wL + rK + qS$. Then,

$$LS = \frac{\sum_i wL_i}{\sum_i p_i Y_i} = \frac{\sum wL_i/C_i \times C_i/C}{\sum p_i Y_i/C_i \times C_i/C} = \frac{\sum \ell_i \omega_i}{\sum \mu_i \omega_i} = \frac{\ell}{\mu}\,. \tag{32}$$

Since a change in software price affects μ as well as ℓ, we need to consider the markup change in equation (31) together with the factor substitution in Proposition 2 when calculating the effect of software-embodied technological change on the aggregate labor income share.

4 Estimation

The micro-level elasticity of substitution can be identified from the relationship between factor income shares and factor prices (Proposition 1). However, estimating this relationship is challenging because of the simultaneous movements in factor inputs and prices driven by the factor bias of technological change, which is typically not observable. To address this challenge, one needs to either control for factor-biased technological changes or employ instruments for factor price changes that are orthogonal to factor-biased technological changes.

In the macro literature, some researchers dealt with this issue by assuming a specific form of factor-biased technological change, such as a log-linear time trend (Antras, 2004; Herrendorf et al., 2015, among others). However, it is unclear whether the assumed functional form adequately controls for all the factor bias. With more disaggregated data, more reasonable instruments become available, such as local labor market impacts of national changes in employment (Raval, 2019; Oberfield and Raval, 2021; Lashkari et al., 2023), amenities (Oberfield and Raval, 2021), and tariffs (de Souza and Li, 2023). However, the estimated elasticity from micro-level data will generally differ from the elasticity at the macro level.

We first estimate the micro-level elasticities of substitution σ_e and σ_s using micro-level data and instruments for relative factor prices. We then aggregate the micro-level elasticities into the macro-level elasticities of substitution, using our theoretical results and the data on the distribution of factor income shares.

We estimate the micro-level elasticities in two different ways, each utilizing a distinct data set with its own strengths and weaknesses. The first approach uses the data on manufacturing establishments from the Korean Economic Census of 2015, encompassing all manufacturing establishments with at least one employee. The census provides information on the geographic location of each establishment, allowing us to use instruments with regional variations. The limitation is that it is one cross-section covering only the manufacturing sector.

The second approach uses firm-level panel data, KISDATA. It covers all sectors, but one limitation is that it only covers firms meeting specific criteria for external auditing. As it lacks information on firms' geographic location, we use instruments with industrial variations over time.

4.1 Cross-sectional Estimates of Micro-level Elasticities

We begin by estimating the micro-level elasticity of substitution between labor and equipment versus software, using the cross-sectional variation in the Korean Economic Census data.

Estimation Strategy In estimating the relationship between relative expenditures on factors and relative factor prices, we follow Oberfield and Raval (2021), with wage differences across regions as our main explanatory variable. Local wages are obtained from the Regional Employment Survey. The Regional Employment Survey is a household-level survey reporting workers' salaries, demographics, educational attainment, and experience. We estimate a residual wage for each person in the manufacturing sector, controlling for education, demographics, and experience, and then aggregate it up to the region level.[14].

We run the following regression across firms:

$$\log \frac{rK_i}{wL_i} = \beta_e \log w_r + \gamma_e X_i + \epsilon_{e,i}, \tag{33}$$

$$\frac{1}{1-k_i} \log \frac{qS_i}{wL_i + rK_i} = \beta_s \log w_r + \gamma_s X_i + \epsilon_{s,i}, \tag{34}$$

where i indexes firms, w_r is the residual wage in region r that firm i belongs to, and X is a set of control variables including three-digit industry fixed effects, firm age, and multi-establishment status. Note that $\beta_e = \sigma_e - 1$ and $\beta_s = \sigma_s - 1$. This specification assumes that all firms face the same cost of capital but different wages across regions.[15]

When using the regional wage variations, endogeneity is an issue if local wages are correlated with unobserved productivity that is not factor neutral. We follow Oberfield and Raval (2021) and Raval (2019) and use Bartik (1991) style shift-share instrument to address this concern. Specifically, we construct a shift-share variable that captures changes in local demand for labor in the service sector and use it as an instrument for the supply of labor for manufacturing plants. The instrument is $Z_r = \sum_{i \in N_s} \omega_{r,i,0} \log(L_{i,t}/L_{i,0})$, where N_s is the set of industries in the service sector, $\omega_{r,i,0}$ is the service industry i's share of employment in region r at time 0, and $L_{i,t}$ is the nationwide employment of industry i in time t. The employment growth is from 2010 to 2015. Because the instrument covers only service industries, we interpret this as a change in the labor supply for the manufacturing firms in the same region competing for the same pool of workers, uncorrelated with the factor-biased productivity of manufacturing firms.[16]

One may wonder about the plausibility of the assumption that firms in the service sector and those in the manufacturing sector hire from the same worker pool in

[14]The unit of a region in our analysis is Si-Gun-Gu, which is an administrative division of South Korea. The average population size of Si-Gun-Gu is 319,200, which is a bit smaller than the average population size of a commuting zone in the US, 443,500.

[15]This is one reason why we focus on equipment rather than structure and equipment in the baseline estimation. It is unlikely that firms located in different regions face the same price of structure.

[16]Oberfield and Raval (2021) employs a measure of local amenities based on climate and geography as an alternative instrument. However, we cannot do the same because measures of climate and geography do not have enough variations in a small country like Korea. Oberfield and Raval (2021) reports that the Bartik instrument and local amenities result in similar estimates of substitution elasticity. In section 4.2, we use a different data set and a different instrumental variable.

a region. Checking for the contribution of each service industry using the Rotemberg weights as in Goldsmith-Pinkham et al. (2020), we find that the research and development industry and the business support service industry account for 93 percent of overall weights and 80 percent of positive weights (Appendix B.4.1). Since it is likely that workers in these two industries can switch to the manufacturing sector more easily than workers in other service industries (for example, health care or education), we view the assumption of the common labor supply pool to be plausible.

As robustness checks, we also estimated a two-stage least squares (TSLS) regression and a limited-information maximum likelihood (LIML) with each industry share as instruments (Goldsmith-Pinkham et al., 2020). In addition, we estimated using an alternative instrument based on industry wage premia suggested by Beaudry et al. (2012).[17] Lastly, regarding the inference of estimation with a shift-share instrument, Borusyak et al. (2021) suggests a shock (industry) level regression, when initial shares are endogenous and identification comes from exogenous shocks. While it is unclear that we should suspect the initial shares to be endogenous, we do a shock-level shift-share IV (SSIV) estimation as a robustness check.

Results Table 2 reports the estimated micro-level elasticities of substitution between labor and equipment (σ_e) and between labor and software (σ_s). We report the OLS estimates in columns 1, and IV estimates in columns 2 to 7. The benchmark result is the estimation with the shift-share instrument in column 2. The elasticity of substitution between labor and equipment mostly lies between 0.3 and 0.7, implying complementarity between labor and equipment. These estimates are in line with those in previous studies of the US data, such as Antras (2004), Herrendorf et al. (2015), Knoblach et al. (2020), Oberfield and Raval (2021), and Raval (2019).

Our novel finding is that the elasticity of substitution between labor and software (σ_s) is greater than one, statistically significantly so in the majority of the specifications, including the benchmark (column 2). It implies that software substitutes for labor and that labor income shares would decrease within firms in response to software-embodied technological change.[18]

[17] Formally, the instrument is $Z_r^{\text{BGS}} = \sum_{i \in N_s} \hat{\omega}_{r,i,t}(\nu_{i,t} - \nu_{i,0})$, where N_s is the set of industries in the service sector, $\hat{\omega}_{r,i,t}$ is the predicted share of region r's employment in industry i, and $\nu_{i,t}$ is the wage premium in industry i in year t. The share $\hat{\omega}_{r,i,t}$ is predicted based on national employment changes as above, and the wage premia $\nu_{i,t}$ are fixed effects from a regression of individual wages on industry dummies indexed by i.

[18] The standard errors are larger for the shock-level regressions (columns 6 and 7). This specification can only utilize the variation across 34 two-digit service industries, with even smaller effective sample size. In a shock-level regression, the effective sample size is the inverse of the Herfindahl index of average industrial employment shares across regions, calculated as $1/\sum_n s_n^2$, where $s_n = \sum_r s_{r,n}$ represents the employment

	OLS	Bartik	BGS	TSLS	LIML	SSIV1	SSIV2
Equipment (σ_e)	0.663	0.600	0.580	0.385	0.375	0.600	-0.164
	(0.084)	(0.153)	(0.222)	(0.070)	(0.098)	(1.051)	(0.815)
Software (σ_s)	1.128	1.620	2.395	1.510	1.519	1.620	2.736
	(0.118)	(0.230)	(0.357)	(0.106)	(0.134)	(0.879)	(0.928)

Table 2: Micro-Level Capital-Labor Substitution: Cross-Sectional Estimates

Column 1 is the OLS estimate. Columns 2 and 3 are IV regressions using shift-share instruments according to Bartik (1991) and Beaudry et al. (2012), respectively. Columns 4 and 5 are IV regression with two-stage least squares (TSLS) and limited-information maximum likelihood (LIML) with each industry share as an instrument. Columns 6 and 7 are the shock (industry) level shift-share IV regressions suggested by Borusyak et al. (2021) with complete shares and incomplete shares, respectively. Standard errors are clustered at the level of 3-digit industry and region for columns 1 to 5. Robust standard errors are reported for columns 6 and 7.

We next do a battery of robustness checks. We first estimate using only the establishments that already existed in 2009 or before ('Old est.' in Table 3), since our instrument has national services employment growth between 2010 and 2015. Second, in the Economic Census, many firms report that they do not hold software assets ($S_i = 0$), which might be measurement errors. We restrict the sample to firms with non-zero software assets ('Positive obs.'). Third, we consider an alternative ordering of nested CES structure that bundles labor and software first instead of labor and equipment ('Alt. order').[19] Fourth, we consider an estimation with tangible (equipment and structure) and intangible capital (software and R&D). Finally, we run the same regression using only standalone establishments, dropping multi-establishment firms ('Standalone'). The results, all using our benchmark Bartik instrument, are in Table 3, which shows $\sigma_e < 1$ and $\sigma_s > 1$ in all cases.

4.2 Panel Estimates of Micro-level Elasticities

In an effort to assess the broader validity of our findings, we estimate the micro-level elasticities of substitution using a different data set and a different identification strategy. To be specific, we utilize firm-level panel data (KISDATA) and the variations in minimum wages as an instrument for labor cost.

share of industry n in region r (Borusyak et al., 2021). Intuitively, when regional employment is dominated by a few industries, there would not be much variation at the shock (industry) level. The effective sample size in our data is only 12.7. Detailed discussion of the shock-level regression is in Appendix B.4.1.

[19]Formally, $V_i = [\{\alpha_i^L(A_L L_i)^{\frac{\sigma_s-1}{\sigma_s}} + \alpha_i^S(A_S S_i)^{\frac{\sigma_s-1}{\sigma_s}}\}^{\frac{\sigma_s(\sigma_e-1)}{(\sigma_s-1)\sigma_e}} + \alpha_i^K(A_K K_i)^{\frac{\sigma_e-1}{\sigma_e}}]^{\frac{\sigma_e}{\sigma_e-1}}$, instead of equation (4).

	Old est.	Positive obs.	Alt. order	Tan/Intan	Standalone
Equipment (σ_e)	0.651	0.674	0.597	0.736	0.595
	(0.162)	(0.318)	(0.154)	(0.165)	(0.300)
Software (σ_s)	1.937	1.438	1.169	2.716	1.502
	(0.279)	(0.404)	(0.198)	(0.429)	(0.387)

Table 3: Robustness Checks for Micro-Level Capital-Labor Substitution

All columns are estimates from IV regressions using our Bartik (1991) instrument. Column 1 only uses establishments established before 2010, and column 2 uses data with strictly positive S_i only. Column 3 is for the alternative CES ordering of the production function. Column 4 uses total tangible vs. intangible capital, instead of equipment vs. software. Column 5 only includes standalone establishments. Standard errors are clustered at the level of three-digit industry and region.

Estimation Strategy To identify the substitution elasticities with panel data, we follow the approach of Chirinko and Mallick (2017), focusing on relatively longer-run variations in factor shares and prices. We first compute three-year moving averages of firms' factor shares and factor prices.[20] We then take differences between adjacent years to remove time-invariant heterogeneity across firms. The regression equations based on Proposition 1 are as follows.

$$\Delta \log(k^*_{i,t}/(1-k^*_{i,t})) = \alpha + (\sigma_e - 1)\Delta \log x^*_{1,i,t} + \beta \Delta \log z^*_{i,t} + \gamma_t + \varepsilon_{i,t} \tag{35}$$

$$\Delta \log(s^*_{i,t}/(1-s^*_{i,t})) = \alpha + (\sigma_s - 1)\Delta \log x^*_{2,i,t} + \beta \Delta \log z^*_{i,t} + \gamma_t + \varepsilon_{i,t} \tag{36}$$

With i indexing firms and j indexing industries, $k_{i,t}$ is $r_{j,t}K_{i,t}/(w_{i,t}L_{i,t})$, and $s_{i,t}$ is $q_{j,t}S_{i,t}/(W_{i,t}L_{i,t} + r_{j,t}K_{i,t})$. Also, $\log x_{1,i,t}$ is $\log w_{i,t}/r_{j,t}$, $\log x_{2,i,t}$ is $(1-k_{i,t})\log w_{i,t} + k_{i,t}\log r_{j,t} - \log q_{j,t}$, $\log z_{i,t}$ is log sales, and γ_t is year dummies. For any variable, the asterisk denotes its three-year moving average.[21]

To address endogeneity concerns, we use changes in the national minimum wage as an instrument for variations in labor cost. The minimum wage is a particularly useful instrumental variable for Korea during our sample period, because the minimum wage increased drastically and unexpectedly. In 2017, the then president was impeached for a reason unrelated with the state of the economy. In the presidential election held to fill the vacancy two months later, the liberal opposition party candi-

[20] Chirinko and Mallick (2017) applied a low-pass filter to the industry-level variables. We are unable to apply the low-pass filter, because our firm-level data is an unbalanced panel.

[21] To obtain the firm-level wage $w_{i,t}$, we divide the labor compensation by the number of employees. For equipment and software rental rates ($r_{j,t}$ and $q_{j,t}$) at the industry level, we use the following imputation, because the National Accounts of Korea does not disaggregate industry-level capital price by type of capital. The rental rate of equipment in industry j is $r_{j,t} = R_{j,t} \times (r_t/R_t)$, where $R_{j,t}$ is the rental rate of total capital in industry j in year t, R_t is the rental rate of total capital in the aggregate, and r_t is the rental rate of equipment in the aggregate. For the rental rate of software in industry j, replace the lower case r's with q's.

date won. The new administration pursued a distinctly pro-labor policy in the labor market, including substantial raises of the minimum wage. It rose by 16.4 percent in 2018 and 10.9 percent in 2019 in nominal terms, when the inflation rate was less than two percent.

Since Korea sets only the national minimum wage, the minimum wage does not vary across industries. To generate industrial variations, we compute two variables for each industry in each year t: (i) the share of workers between the minimum wage in year t and the minimum wage in year $t+1$, and (ii) the share of workers reporting hourly wages below the minimum wage in year t. We then run instrumental variable regressions, using the current and one-year lagged values of these two variables as instruments for the relative factor prices ($\Delta \log x_{i,t}$). In an alternative specification, we control for firms' sales to capture changes in market demand.

Results Table 4 reports the estimation results. The elasticity of substitution between labor and equipment ranges from 0.603 to 0.627 across different specifications (columns 1 to 2), indicating complementarity between labor and equipment. On the other hand, the substitution elasticity between labor and software is invariably greater than one, confirming our previous findings from cross-sectional estimations. The patterns remain robust, when (i) we reverse the ordering of factors in the production function as in footnote 19 ('Alt. order'), (ii) we use tangible and intangible assets instead of equipment and software ('Tan/Intan'), and (iii) we replace firm-level wages with the industry average of hourly male wages controlling for education and experience ('Residual wage').

	Baseline		Alt. order	Tan / Intan	Residual wage
Equipment (σ_e)	0.603	0.627	0.627	0.047	0.396
	(0.099)	(0.097)	(0.100)	(0.079)	(0.089)
Software (σ_s)	1.482	1.479	1.590	1.672	1.798
	(0.259)	(0.259)	(0.386)	(0.177)	(0.478)
sales control	✓		✓	✓	✓

Table 4: Micro-Level Capital-Labor Substitution: Panel Estimates

All columns are estimates from two-stage least squared regressions using minimum wage instruments. Specifically, instrumental variables are the current and one-year lagged values of the share of workers between the minimum wage in year t and the minimum wage at time $t+1$ by industry and the share of workers reported wage below the minimum wage by industry. Standard errors are clustered at the firm level.

From the micro-level estimates derived from distinct datasets and identification strategies, we conclude that software substitutes for labor, whereas equipment complements labor. This difference in the elasticity of substitution suggests that software may play a more crucial role in the decline of the aggregate labor share.

4.3 Aggregation

We now compute the aggregate elasticities of substitution, which determine the impact of factor prices on aggregate factor income shares and markup. Applying the theory in Section 3, we derive the aggregate elasticities $\bar{\sigma}_e$ and $\bar{\sigma}_s$ as in Proposition 2, combining our estimates of the micro-level elasticities of substitution and the moments from the joint distribution of factor shares and markups in the microdata.

Data and Estimated Parameters For our baseline aggregation result, we use the firm-level panel KISDATA from 2003 to 2018. To compute the reallocation parameters, we need to estimate markups (μ_i) and price pass-through (b_i) at the firm level. A firm's markup is given by the ratio of revenue to expenditure on a variable input, multiplied by the output elasticity with respect to the factor. For example, with labor as a variable input, the markup μ_i of firm i is

$$\mu_i = \frac{p_i y_i}{wL_i} \times \alpha_i \, , \tag{37}$$

where $p_i y_i$ is revenue and α is the output elasticity with respect to labor. Because the output elasticity is not directly observed in the data, the literature suggests several methods of estimating markup (Baqaee and Farhi, 2019; De Loecker et al., 2020; Edmond et al., 2018, among others). We use the ratio of revenue to total cost as our baseline estimate, inferring α from the firm's cost minimization condition, $\alpha_i = wL_i/(wL_i + rK_i + qS_i + vM_i)$. Further details on the cases with alternative markup measures are in Appendix B.2.3.

From the estimated markups, we compute each firm's price pass-through (b_i) using the relationship between markup and pass-through given by the demand aggregator in Klenow and Willis (2016):

$$H(Y_i/Y) = 1 + (\sigma - 1)\exp(1/\nu)\nu^{\frac{\sigma}{\nu}-1}\left[\Gamma\left(\frac{\sigma}{\nu}, \frac{1}{\nu}\right) - \Gamma\left(\frac{\sigma}{\nu}, \frac{(Y_i/Y)^{\frac{\nu}{\sigma}}}{\nu}\right)\right].$$

With this functional form, $\epsilon(x) = -\frac{h'(x)x}{h(x)} = \sigma h(x)^{-\frac{\nu}{\sigma}}$ and hence $\frac{\epsilon'(x)x}{\epsilon(x)} = -\frac{\nu}{\sigma}\frac{h'(x)x}{h(x)} = \frac{\nu}{\sigma}\epsilon(x)$. Since $\mu = \frac{\epsilon}{\epsilon - 1}$, we obtain the following relationship between the markup μ_i

and the price pass-through b_i:

$$b_i = \frac{1}{1 - \frac{\mu' x}{\mu}} = \frac{1}{1 + \mu_i \frac{\nu}{\sigma}} \,. \tag{38}$$

The value of the super-elasticity ν/σ is set to 0.11, from the empirical relationship between firms' sales shares and markups, as in Edmond et al. (2018).

From the various micro-level elasticity estimates in Sections 4.1 and 4.2, we select $\sigma_s = 1.6$ and $\sigma_e = 0.6$, the Bartik instrument estimates in Table 2, which are not much different from other estimates. We also need to set the elasticity of substitution between value added and material input (σ_m). We estimate σ_m using our cross-sectional approach (Section 4.1), which results in $\sigma_m = 1$.[22] With $\sigma_m = 1$, the adjustment terms reflecting the discrepancy between value-added and sales disappear ($\sigma_m - 1 = 0$) in Propositions 2 and 3.[23]

Lastly, we take five-year moving averages to smooth the reallocation parameters. Details on the construction of the reallocation parameters are in Appendix B.

Results In Table 5, we report the macro-level elasticities ($\bar{\sigma}$, first column) and decompose the total change in relative factor income shares ($\bar{\sigma} - 1$, second column) into the within-firm change ($\sigma - 1$, third column) and the remaining part that is due to reallocation ($\bar{\sigma} - \sigma$, fourth column). For example, in response to a one percent increase in labor cost, the expenditure on equipment relative to labor (rK/wL) changes by $\sigma_e - 1$ percent within firms and $\bar{\sigma}_e^w - \sigma_e$ percent due to reallocation, with a total change of $\bar{\sigma}_e^w - 1$ percent in the aggregate. As shown in Section 3.2.2, this total change in response to a change in wage is different from the one in response to a change in the cost of equipment, $\bar{\sigma}_e^r - 1$. In the last two columns of Table 5, we report the distributional moments that determine the magnitude of reallocation. We hold fixed the micro-level elasticities over time, but the macro-level elasticities do vary over time because the joint distribution of factor shares and markups evolves over time. In the table, we show the values for 2005 and 2015. The time series are shown in Figure 5, but we do not find a clear time trend in the macro-level elasticities.

We first note that the macro-level elasticities of substitution between equipment and labor are larger than the micro-level elasticity ($\bar{\sigma}_e > \sigma_e$). In fact, in response to

[22] The estimation equation is $(1/\ell_i) \log m_i/(1 - m_i) = \beta_m \log w_r + \gamma_m X + \epsilon_{m,i}$, which implies $\sigma_m = \hat{\beta}_m + 1$. The OLS and the IV estimation with the shift-share instrument give σ_m estimates of 0.8 and 1.3, respectively. This is similar to what Oberfield and Raval (2021) report in their analysis of US data.

[23] As Castro-Vincenzi and Kleinman (2023) show, within a firm, a rise in material costs reduces its labor share when labor and material input are complements ($\sigma_m < 1$) and markups are positive. In our analysis, σ_m affects the labor share through reallocation across firms as well. If $\sigma_m < 1$, the impact of capital-embodied technological change is attenuated, resulting in less reallocation.

changes in labor costs, equipment even substitutes for labor ($\bar{\sigma}_e^w > 1$) in the aggregate, even though the micro-level elasticity is well below one. However, with respect to changes in the price of equipment, equipment mostly complements labor ($\bar{\sigma}_e^r < 1$). This discrepancy results from the difference in the magnitude of reallocation with respect to the two factor prices ($\bar{\epsilon}_e^w > \bar{\epsilon}_e^r$). It reflects the fact that firms facing higher demand elasticities tend to have more extreme labor expenditure shares (either very high or very low) in the data (Proposition 2).

Reallocation plays a relatively minor role in the macro-level elasticity of substitution between software and labor ($\bar{\sigma}_s$). Accordingly, this macro-level elasticity remains close to the micro-level elasticity of substitution between software and labor, no matter which factor price changes. Both the weight on the reallocation and the magnitude of the reallocation are smaller for the software-labor elasticity than for the equipment-labor elasticity ($\xi < \chi$ and $\bar{\epsilon}_s < \bar{\epsilon}_e$). The relatively small $\bar{\epsilon}_s^q$ implies that firms that use

	Year	Elasticity ($\bar{\sigma}$)	Total Change	Within Change	Re-allocation	Distributional Moments	
Factor substitution (eqp.)			$\bar{\sigma}_e - 1$	$\sigma_e - 1$	$\bar{\sigma}_e - \sigma_e$	χ	$\bar{\epsilon}_e$
$\bar{\sigma}_e^w$	2005	1.173	0.173	-0.400	0.573	0.289	4.560
	2015	1.159	0.159	-0.400	0.559	0.334	3.707
$\bar{\sigma}_e^r$	2005	0.834	-0.166	-0.400	0.234	0.289	1.925
	2015	1.036	0.036	-0.400	0.436	0.334	2.925
Factor substitution (sft.)			$\bar{\sigma}_s - 1$	$\sigma_s - 1$	$\bar{\sigma}_s - \sigma_s$	ξ	$\bar{\epsilon}_s$
$\bar{\sigma}_s^q$	2005	1.561	0.561	0.600	-0.039	0.068	1.048
	2015	1.599	0.599	0.600	-0.001	0.033	2.295
$\bar{\sigma}_s^r$	2005	1.979	0.979	0.600	0.379	-0.526	0.880
	2015	1.434	0.434	0.600	-0.166	0.246	0.846
$\bar{\sigma}_s^w$	2005	1.468	0.468	0.600	-0.132	-0.007	0.920
	2015	1.600	0.600	0.600	0.000	0.513	0.326
Changes in markup			$\bar{\sigma}_\mu - 1$	$\bar{b} - 1$	$\bar{\sigma}_\mu - \bar{b}$	η	$\bar{\epsilon}_\mu$
$\bar{\sigma}_\mu^q$	2005	0.503	-0.497	-0.423	-0.074	0.198	0.637
	2015	0.677	-0.323	-0.245	-0.078	-0.003	1.430
$\bar{\sigma}_\mu^r$	2005	0.699	-0.301	-0.240	-0.062	-0.043	2.199
	2015	0.690	-0.310	-0.210	-0.100	0.017	-5.228
$\bar{\sigma}_\mu^w$	2005	0.791	-0.209	-0.223	0.014	0.013	1.831
	2015	0.804	-0.196	-0.217	0.021	-0.003	-7.319

Table 5: Macro-level Elasticities and Distributional Moments

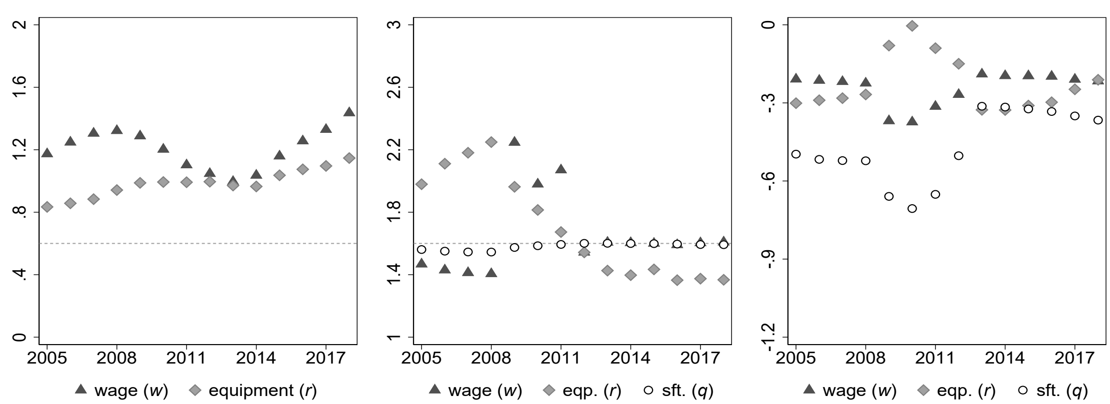
The macro-level elasticities are based on Propositions 2 and 3 and the distributional moments from the joint distribution of k_i, ℓ_i, s_i, κ_i, m_i, θ_i, ω_i, γ_i, ϵ_i, and b_i in KISDATA. The micro-level elasticities are the Bartik IV estimates in Table 2.

software intensively does not respond much to the change in the price of software. This is because such software-intensive firms tend to have high markups (and low demand elasticities) in the data.

In fact, reallocation can even counteract the substitution between labor and software within firms. Normally, for example, an increase in the labor cost reduces the market share of labor-intensive firms, further reducing the aggregate labor share. However, when labor-intensive firms also have relatively high software shares ($\xi^w < 0$) or when the sales of such labor-intensive firms with high software intensities respond sensitively to the shock ($\bar{\epsilon}_s^w < 1$), reallocation can result in a lower aggregate software share. As discussed in Section 3.2.2, such an outcome is not possible when there are only two factors.

Even if reallocation had a small effect in terms of factor substitution, it could still have a sizeable effect on factor shares through endogenous changes in markup. Changes in factor prices affect not only within-firm markup but also the aggregate markup through reallocation, which in turn affect the aggregate income shares of all factors.

The bottom panel of Table 5 shows that the aggregate markup decreases (increases) when factor prices increase (decrease), $\bar{\sigma}_\mu - 1 < 0$. Because of the incomplete price pass-through in our specification ($b_i < 1$), on average, within-firm markup will decrease in response to higher costs ($\bar{b} < 1$, third column). Reallocation across firms can either reinforce or counteract this within-firm markup change, depending on the joint



(a) Equipment ($\bar{\sigma}_e$) (b) Software ($\bar{\sigma}_s$) (c) Markup Change ($\bar{\sigma}_\mu - 1$)

Fig. 5: Macro-level Elasticities

The dotted lines in panels (a) and (b) are the micro-level elasticity estimates. The macro-level elasticities are based on Propositions 2 and 3 and the distributional moments from the joint distribution of k_i, ℓ_i, s_i, κ_i, m_i, θ_i, ω_i, γ_i, ϵ_i, and b_i in KISDATA.

distribution of factor shares and markups across firms in the data. The weights (η, fifth column) are proportional to the correlation between factor shares and markups. In our data, markups are not significantly correlated with labor shares or equipment shares. On the other hand, they are positively correlated with software shares so that reallocation reinforces the effect of the within-firm markup changes in response to software price changes. As a result, of the three factor prices, the software price has the largest effect on the aggregate markup.

5 Macro-level Elasticities and Aggregate Labor Share

The estimated elasticities of substitution in Section 4 suggest that it is software, not equipment, whose declining price contributed more to the fall in the aggregate labor share. We now quantify the respective contribution. Proposition 4 summarizes the impact of capital-embodied (software and equipment) technological changes on the aggregate labor share.[24]

Proposition 4 (Change in the aggregate labor share) *The impact of capital-embodied technological changes on the aggregate labor share is given by:*

$$LS_t - LS_{t-1} = \overline{LS}_t \times \left[-\left(\bar{s}_t(\bar{\sigma}^q_{s,t} - 1) - \bar{\bar{s}}_t(\bar{\sigma}^q_{\mu,t} - 1) \right) \ln \frac{1/q_t}{1/q_{t-1}} \right.$$
$$\left. - \left(\bar{e}_t(\bar{\sigma}^r_{e,t} - 1) - \bar{e}_t\bar{s}_t(\bar{\sigma}^r_{s,t} - 1) - \bar{\bar{\kappa}}_t(\bar{\sigma}^r_{\mu,t} - 1) \right) \ln \frac{1/r_t}{1/r_{t-1}} \right], \tag{39}$$

where $\bar{x}_t \equiv \frac{x_t + x_{t-1}}{2}$.

Proof *In Appendix A.* ∎

Given the aggregate elasticities in Table 5, equation (39) implies that software-embodied technological change ($d \ln 1/q > 0$) reduces the aggregate labor income share through both factor substitution ($\bar{\sigma}^q_s > 1$) and markup changes ($\bar{\sigma}^q_\mu < 1$). On the other hand, equipment-embodied technological change ($d \ln 1/r$) has countervailing effects. In terms of factor substitution, it always increases the labor share within a firm because equipment complements labor ($\sigma_e < 1$). However, reallocation may or may not counteract the pattern of within-firm factor substitution ($\bar{\sigma}^r_e \gtrless 1$). The responses of markup invariably reduces the aggregate labor share ($\bar{\sigma}^r_\mu < 1$).

[24]Our production function in effect assumes that other types of capital have a unitary elasticity of substitution with respect to labor and hence do not affect the labor share. See proof of Proposition 4 for details. Since the prices of equipment and software fell much more than those of other capital types in the data, our results do not hinge on this assumption. Figure 2 supports this assumption.

Data To implement the decomposition in equation (39), we construct the aggregate labor income share LS_t, aggregate equipment income share e_t, and aggregate software income share s_t from the National Accounts as described in Section 2.[25] Software and equipment-embodied technological change is represented by five-year moving averages of the inverse of the price of software and equipment investment relative to consumption, as in Section 2. The decomposition starts in 1990, when Korea's aggregate labor income share started to decline in a sustained manner. As the cross-sectional moments are available only after 2003, we impute the values between 1990 and 2002 from the 2003 values. We note that the aggregate elasticities do not have clear time trends (Figure 5). Further details are in Appendix B.

Results Table 6 and Figure 6 report by how much capital-embodied technological change affected the aggregate labor income share between 1990 and 2018 in Korea. The aggregate labor share declined by 4.4 percentage points in the data.

Using the estimated macro-level elasticities and the software price time series, we find that software-embodied technological change accounts for 2.9 p.p. or 66.9 percent of the overall decline in the labor share (first column, top panel). Of the 2.9 p.p., 1.4 p.p. (second column) is due to factor substitution and the other 1.5 p.p. (fifth column) is due to the rise in aggregate markup in response to the falling software price, a near-even split.

The decline in the labor share through factor substitution is entirely (1.5 out of 1.4 p.p.) attributed to within-firm substitution of software for labor (governed by the micro-level elasticity σ_s) and not at all to reallocation across firms (third and fourth columns). As for the markup increase in response to the falling software price, within-firm and between-firm adjustments play comparable roles (0.8 and 0.7 p.p. respectively out of 1.5 p.p.). One may think that reallocation overall has a minor effect on the decline of the labor share, but it should be noted that the within-firm markup increase comes with a rise in market share, which entails reallocation. In fact, as in Kehrig and Vincent (2021), reallocation can be broken into three components: (i) large firms reducing their labor share, (ii) firms with a low labor share growing rapidly, and (iii) firms simultaneously gaining market shares and decreasing their labor shares. Their study of the US manufacturing plants found that the decline in the aggregate labor share was mostly driven by those plants that expanded rapidly while reducing their labor share—that is, (iii) above. Our decomposition is different from theirs, but

[25] We use the same values for $\tilde{s}$ as for s and $\tilde{\kappa}$ as for κ. Formally, $\tilde{s} = \sum \omega_i (s_i(1-m_i) + m_i \alpha_p^q) \approx \sum \omega_i s_i = s$ and $\tilde{\kappa} = \sum \omega_i (\kappa_i(1-m_i) + m_i \alpha_p^r) \approx \sum \omega_i \kappa_i = \kappa$.

- Total change in labor share in the data: -0.044

- Effects of Software-embodied Technological Change ($\Delta \ln 1/q$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.029	-0.014	-0.015	+0.001	-0.015	-0.008	-0.007
(% explained)	(66.9)	(31.9)	(33.0)	(-1.2)	(35.0)	(19.0)	(16.1)

- Effects of Equipment-embodied Technological Change ($\Delta \ln 1/r$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.004	+0.004	+0.014	-0.010	-0.009	-0.008	-0.001
(% explained)	(10.2)	(-9.5)	(-32.8)	(23.3)	(19.8)	(17.4)	(2.3)

Table 6: Effects of Capital-Embodied Technological Change on the Labor Share

The decomposition is for the 1990–2018 period. The overall effect is calculated using the macro-level elasticities ($\bar{\sigma}$), and the within effect is computed using the micro-level elasticities (σ). The between effect is the difference between the overall and the within effects. Percent explained of the labor share decline in the data is in parentheses.

the importance of the within-firm factor substitution and the within-firm markup increase in our result is consistent with firms increasing market shares while simultaneously reducing their labor shares. More important, our result suggests that software-embodied technological change may well be the cause of the pattern of reallocation observed both in the US data and in the Korean data.

The left panel of Figure 6 plots the cumulative impact of the fall in the software price on the aggregate labor share through within/between factor substitution and within/between markup increases.

The effect of equipment-embodied technological change is shown in the bottom panel of Table 6. Using the estimated macro-level elasticities and the equipment price time series, we obtain a 0.4 percentage point decrease in the labor share (or 10 percent of the actual decrease) from equipment-embodied technological change. Behind this small number are several conflicting forces. Within-firm substitution pushes up the labor share by 1.4 p.p. (third column) because $\sigma_e < 1$, but the reallocation toward low labor share firms goes against it, negating 1.0 (fourth column) of the 1.4 p.p. increase. Factor substitution overall raises the labor share by 0.4 p.p. when the equipment price falls. In addition, the falling equipment price raises markups and reduces the labor share by 0.9 p.p. (fifth column), mostly through what happens within firms (sixth column). In sum, because of the heterogeneous markups, equipment-embodied

(a) Software Contribution

(b) Equipment. Contribution

Fig. 6: Effects of Capital-Embodied Technological Change on the Labor Share

The decomposition is for the 1990–2018 period. The overall effect is calculated using the macro-level elasticities ($\bar{\sigma}$), and the within effect is computed using the micro-level elasticities (σ). The between effect is the difference between the overall and the within effects. Percent explained of the labor share decline in the data is in parentheses.

technological change contributes to a small reduction in the labor share, even though equipment and labor are found to be complements at both the micro and the macro levels.

Put together, software-embodied and equipment-embodied technological change accounts for 77 percent of the decline in the aggregate labor share between 1990 and 2018 in Korea. Software-embodied technological change is the key driver (67 of the 77 percent), and variable markups are crucial for a comprehensive understanding of the overall impact of capital-embodied technological change. Although not reported here, when we simply lump together equipment and software as one capital bundle in a two-factor production function, we find this capital bundle and labor are complements: This simpler specification is a non-starter for thinking about the decline of the labor share in the face of falling capital prices. Furthermore, even with software and equipment as two separate factors of production, When we use the standard CES aggregator with constant markup, software-embodied technological change accounts for less than one-third of the decline in the aggregate labor share.

In Appendix C, we provide additional results as robustness checks. First, we provide decomposition results using the aggregate elasticities and distributional moments from the manufacturing census data. Second, we consider alternative values of pass-through (b_i) based on the empirical relationship in Baqaee et al. (2023).[26] In both

[26] Baqaee et al. (2023) suggest that the Klenow-Willis specification produces too little variation in price

cases, we obtain qualitatively similar results: The equipment-labor substitution elasticity is larger at the macro level than at the micro level (but still less than one), while the macro-level software-labor substitution elasticity remains close to its micro-level counterpart. In addition, the aggregate markup is much more responsive to software prices than other factor prices.

6 Conclusion

In this paper, we establish that the capital-labor substitution elasticity is different across types of capital. The equipment-labor substitution elasticity is less than one, consistent with the micro-level findings in the literature, but the software-labor substitution elasticity is greater than one, a novel finding. This distinction is important for understanding the decline of the aggregate labor income share over time.

Our focus on software connects well with (and nicely complements) the three leading explanations of the labor share decline. First, our results support the literature arguing that technological change embodied in capital reduced the labor share since the 1980s (Karabarbounis and Neiman, 2013). We clarify that it is software rather than equipment that substitutes for labor, both at the micro and the macro levels. Second, our results reinforce the findings of (Koh et al., 2020) that intangible capital shares, rather than tangible capital shares, rose at the expense of labor shares in the aggregate. We not only estimate the elasticities of substitution to quantify the role of capital-embodied technological change, but also establish the importance of the heterogeneous, variable markups in pushing down the labor share as a result. In fact, the effect of the rising markups is as large as the factor substitution itself. In other words, comparing the factor income shares of labor and capital alone will underestimate the true impact of software-embodied technological change by half. Third, the literature has documented how reallocation across firms contributed to the decline of the aggregate labor share (Autor et al., 2020; De Loecker et al., 2020; Kehrig and Vincent, 2021). We find that firms with high software intensity generally have low labor shares and high markups in the data. This means that software-embodied technological change can be the causal force behind the reallocation toward firms with high markups and low labor shares in the data, bringing down the aggregate labor income share.

The natural next step is to ask how such technological change affected different workers differently and hence the overall income inequality. Because of the limited scope of worker heterogeneity in our data, we were not able to assess how equipment

pass-through, potentially understating the magnitude of reallocation.

and software interact with workers of different skill levels. As previously mentioned, our calibration exercise using macro data in the spirit of Krusell et al. (2000) finds that equipment complements both high-skill and low-skill workers, but complements high-skill workers more. Software, on the other hand, substitutes for both high-skill and low-skill workers, but substitutes for low-skill workers more. It suggests that capital-embodied technological change raises skill premium and reduces the income share of low-skill workers. At a more disaggregated level, Aum (2020) documents that workers in middle-skill occupations tend to use equipment more, while those in high-skill occupations tend to use software more in the US O*NET data. We think that a richer framework that allows for multiple types of capital and heterogeneous workers—for example, an enhanced version of the model in Aum et al. (2018)—is a promising avenue for future research on the distributional consequences of technological progress.

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Appendix A Proofs

Proof of Proposition 1 From equations (8) and (9), we have the relative price between wage and equipment price as:

$$\frac{w}{r} = \frac{\alpha_i^L}{\alpha_i^K}\left(\frac{A_L}{A_K}\right)^{\frac{\sigma_e-1}{\sigma_e}}\left(\frac{K_i}{L_i}\right)^{\frac{1}{\sigma_e}}.$$

Taking logs, we have

$$(\sigma_e - 1)\ln\frac{w}{r} = \sigma_e \ln\frac{\alpha_i^L}{\alpha_i^K} + (\sigma_e - 1)\ln\frac{A_L}{A_K} + \ln\frac{rK_i}{wL_i}. \tag{A.1}$$

Differentiating equation (A.1) with A_L/A_K fixed and using the definition of k_i, we obtain equation (15), which is the first part of Proposition 1.

Similarly, from equations (5), (8), (9), and (13), we have $\phi_i = p_i(Y_i/V_i)^{1/\sigma_m}(V_i/X_i)^{1/\sigma_s}$. Dividing it by (10), we obtain

$$\frac{\phi_i}{q} = \frac{1}{\alpha_i^S}\left(\frac{1}{A_S}\right)^{\frac{\sigma_S-1}{\sigma_S}}\left(\frac{S_i}{X_i}\right)^{\frac{1}{\sigma_S}}.$$

Taking logs, we have

$$(\sigma_s - 1)\ln\frac{\phi_i}{q} = \sigma_s \ln\frac{1}{\alpha_i^S} + (\sigma_s - 1)\ln\frac{1}{A_S} + \ln\frac{qS_i}{\phi_i X_i}. \tag{A.2}$$

Differentiating equation (A.2) with A_S fixed, we derive

$$\sigma_s - 1 = \frac{d\ln\frac{qS_i}{\phi_i X_i}}{d\ln\frac{\phi_i}{q}}. \tag{A.3}$$

From the definition of ϕ_i in equation (13),

$$d\ln\phi_i = \frac{1}{1+\left(\frac{\alpha_i^K}{\alpha_i^L}\right)^{\sigma_e}\left(\frac{rA_L}{wA_K}\right)^{1-\sigma_e}} d\ln w + \frac{1}{1+\left(\frac{\alpha_i^L}{\alpha_i^K}\right)^{\sigma_e}\left(\frac{wA_K}{rA_L}\right)^{1-\sigma_e}} d\ln r. \tag{A.4}$$

Since $\left(\frac{r\alpha_i^L}{w\alpha_i^K}\right)^{\sigma_e}\left(\frac{A_L}{A_K}\right)^{\sigma_e-1}\frac{w}{r} = \frac{wL_i}{rK_i}$ from equations (8) and (9), inserting it into equation (A.4) and using the definition of k_i, we obtain

$$d\ln\phi_i = (1-k_i)d\ln w + k_i d\ln r.$$

Finally, we have $\phi_i X_i = wL_i + rK_i$ from equations (5), (8), (9), and (13). Equation (A.3) becomes

$$\sigma_s - 1 = \frac{d\ln\frac{qS_i}{wL_i+rK_i}}{(1-k_i)d\ln\frac{w}{q} + k_i d\ln\frac{r}{q}},$$

which is the second part of Proposition 1. ∎

Proof of Corollary 1 Totally differentiating equation (A.1) and setting $d\ln w = d\ln r = 0$, it is straightforward to see the first part of Corollary 1. Totally differentiating equation (A.3) and setting $d\ln q = 0$ and following the same steps in the proof of Proposition 2, the second part of Corollary 1 follows. ∎

Proof of Proposition 2 Let the cost function of establishment i be $c_i \equiv c(Y_i, w, r, q, p) = wL_i + rK_i + qS_i + pM_i$, and $c = \sum_i c_i$. Firstly, consider $d\ln w > 0$, $d\ln r = 0$, and $d\ln q = 0$.

Equipment-labor substitution From the definition of $\bar{\sigma}_e^w$ (Definition 1),

$$\bar{\sigma}_e^w - 1 = \frac{d\ln\frac{rK}{wL}}{d\ln w} = \frac{d\ln\frac{k}{1-k}}{d\ln w} = \frac{1}{1-k}\frac{d\ln k}{d\ln w} = \frac{1}{k(1-k)}\frac{dk}{d\ln w}. \tag{A.5}$$

From the definition of $k \equiv \sum \theta_i k_i$,

$$dk = \sum_i \theta_i dk_i + \sum_i k_i d\theta_i. \tag{A.6}$$

From equation (15) in Proposition 1, we have

$$dk_i = (\sigma_e - 1)k_i(1-k_i)d\ln w. \tag{A.7}$$

Denoting $\chi \equiv \frac{\sum_i (k_i - k)^2}{k(1-k)}$, we get $1 - \chi = \frac{\sum_i \theta_i(1-k_i)k_i}{k(1-k)}$. Hence, equations (A.5)–(A.7) imply that $\frac{\sum_i \theta_i dk_i}{k(1-k)d\ln w} = (1-\chi)(\sigma_e - 1)$ and

$$\bar{\sigma}_e^w - 1 = (1-\chi)(\sigma_e - 1) + \frac{1}{k(1-k)}\sum_i k_i \frac{d\theta_i}{d\ln w}. \tag{A.8}$$

We now turn to $\sum_i k_i \frac{d\theta_i}{d\ln w}$. Following Oberfield and Raval (2021), we repeatedly use that $\sum_i x d\theta_i = 0$ for any x not variying across i. For example, $\sum_i k_i \frac{d\theta_i}{d\ln w} = \sum_i (k_i - k)\frac{d\theta_i}{d\ln w} = \sum_i (k_i - k)\theta_i \frac{d\ln\theta_i}{d\ln w}$. We will also use that $\sum_i x(k_i - k)\theta_i = 0$ for any x not varying across i.

Since $\theta_i = \frac{(1-s_i)(1-m_i)c_i}{(1-s)(1-m)c}$, $\theta_i d\ln\theta_i = d\ln(1-s_i) + d\ln(1-m_i) + d\ln c_i - d\ln(1-s)(1-m)c$. Note that $\sigma_m - 1 = \frac{d\ln m_i/(1-m_i)}{d\ln \varphi_i/v}$, where φ_i refers to the price of labor-equipment-software composite (i.e. $\varphi_i V_i = wL_i + rK_i + qS_i$). By Shephard's lemma, $\frac{d\ln\varphi_i}{d\ln w} = \ell_i$. Also, we denote labor content of material as $\alpha_v^w \equiv \frac{d\ln v}{d\ln w}$, and assume that labor content of material is the same as labor content in the final good (i.e. $\alpha_v^w = \alpha_p^w \equiv \frac{d\ln p}{d\ln w}$). Combining, we have $\frac{d\ln(1-m_i)}{d\ln w} = -\frac{dm_i}{(1-m_i)d\ln w} = m_i(\ell_i - \alpha_p^w)(1-\sigma_m)$. Similarly, from $\frac{d\ln s_i/(1-s_i)}{d\ln\phi_i/q}$, we have $\frac{d\ln(1-s_i)}{d\ln q} = -\frac{ds_i}{1-s_i} = s_i(1-k_i)(1-\sigma_s)$.

Taking together, and using that $\sum_i (k_i - k)\theta_i d\ln(1-s)(1-m)c = 0$, we have

$$\sum_i k_i \frac{d\theta_i}{d\ln w} = \sum_i (k_i - k)\theta_i \left[\frac{d\ln(1-s_i)}{d\ln w} + \frac{d\ln(1-m_i)}{d\ln w} + \frac{d\ln c_i}{d\ln w}\right]$$

$$= \sum_i (k_i - k)\theta_i \left[s_i(1-k_i)(1-\sigma_s) + m_i(\ell_i - \alpha_p^w)(1-\sigma_m) + \frac{d\ln c_i}{d\ln w} \right], \tag{A.9}$$

where $\alpha_p^w \equiv \frac{d\ln p}{d\ln w} = \frac{d\ln v}{d\ln w}$.

Differentiating cost function $c_i = c(Y_i, w, r, q, p)$, $d\ln c_i = \frac{c_{Y,i}Y_i}{c_i} d\ln Y_i + \frac{c_{w,i}w}{c_i} d\ln w$. From Shephard's lemma, $c_{w,i}w/c_i = \ell_i(1-m_i) + m_i\alpha_p^w$. Constant returns to scale implies $\frac{c_{Y,i}Y_i}{c_i} = 1$. Hence,

$$\frac{d\ln c_i}{d\ln w} = \frac{d\ln Y_i}{d\ln w} + \ell_i(1-m_i) + m_i\alpha_p^w. \tag{A.10}$$

Substituting equation (A.10) into (A.9),

$$\begin{aligned}
\sum_i k_i \frac{d\theta_i}{d\ln w} &= \sum_i (k_i - k)\theta_i \left[s_i(k_i - 1)\sigma_s - m_i(\ell_i - \alpha_p^w)\sigma_m + (1-k_i) + \frac{d\ln Y_i}{d\ln w} \right] \\
&= \sum_i (k_i - k)\theta_i \left[(k_i - 1)\zeta^w\sigma_s - \bar{m}_e^w((1-k_i)(1-s_i) - \alpha_p^w)\sigma_m + (1-k_i) + \frac{d\ln Y_i}{d\ln w} \right] \\
&= \sum_i (k_i - k)\theta_i \left[(k_i - 1)\zeta^w\sigma_s - \bar{m}_e^w((1-k_i)(1-\zeta^w) - \alpha_p^w)\sigma_m + (1-k_i) + \frac{d\ln Y_i}{d\ln w} \right] \\
&= \sum_i (k_i - k)\theta_i \left[(k_i - k)(\zeta^w\sigma_s + (1-\zeta^w)\bar{m}_e^w\sigma_m - 1) + \frac{d\ln Y_i}{d\ln w} \right] \\
&= k(1-k)\chi(\zeta^w\sigma_s + (1-\zeta^w)\bar{m}_e^w\sigma_m - 1) + \sum_i (k_i - k)\theta_i \frac{d\ln Y_i}{d\ln w}.
\end{aligned} \tag{A.11}$$

Here, we denote

$$\zeta^w \equiv \frac{\sum_i (k_i - k)(1-k_i)\theta_i s_i}{\sum_i (k_i - k)(1-k_i)\theta_i}, \text{ and } \bar{m}_e^w \equiv \frac{\sum_i (k_i - k)(\ell_i - \alpha_p^w)\theta_i m_i}{\sum_i (k_i - k)(\ell_i - \alpha_p^w)\theta_i}$$

Substituting equation (A.11) into (A.8), we get

$$\bar{\sigma}_e^w = (1-\chi)\sigma_e + \chi\left(\zeta^w\sigma_s + (1-\zeta^w)\bar{m}_e^w\sigma_m\right) + \frac{1}{k(1-k)} \sum_i (k_i - k)\theta_i \frac{d\ln Y_i}{d\ln w}. \tag{A.12}$$

Now we turn to $\frac{d\ln Y_i}{d\ln w}$. We have aggregate Y that satisfies $1 = \sum_i H(Y_i/Y)$. And cost minimization implies $p_i/p = H'(Y_i/Y)$. Denoting the inverse of H' with h, we have

$$d\ln Y_i/Y = -\epsilon_i d\ln p_i/p, \tag{A.13}$$

where $\epsilon_i \equiv -\frac{h'(p_i/p)p_i/p}{h(p_i/p)}$. The optimal markup will be $\mu(p_i/p) = \frac{\epsilon(p_i/p)}{\epsilon(p_i/p)-1}$. Optimal pricing implies $p_i = \mu(p_i/p)c_{Y,i}$.

We first define firm i's local rate of price pass-through ($b_i \equiv \frac{d\ln p_i}{d\ln mc_i} \equiv \frac{d\ln p_i}{d\ln c_{Y,i}}$). Differentiating optimal pricing condition, $d\ln p_i = \frac{\mu' \times p_i/p}{\mu} d\ln p_i + d\ln c_{Y,i}$. Rearranging, we get $b_i \equiv \frac{d\ln p_i}{d\ln c_{Y,i}} = \frac{1}{1-\frac{\mu' \times p_i/p}{\mu}}$.

Note that a change in wage would also affect aggregate price ($\alpha_p^w \equiv \frac{d\ln p}{d\ln w}$). From equation (A.13), we know that $d\ln Y_i/Y$ is related to $d\ln p_i/p$. Since optimal pricing implies $p_i/p = \mu(p_i/p)c_{Y,i}/p$, differentiating gives

$$\frac{d\ln p_i/p}{d\ln w} = \frac{\mu'(p_i/p)p_i/p}{\mu(p_i/p)}\frac{d\ln p_i/p}{d\ln w} + \frac{d\ln c_{Y,i}}{d\ln w} - \frac{d\ln p}{d\ln w}.$$

Constant returns to scale and Shephard's lemma implies $\frac{d\ln c_{Y,i}}{d\ln w} = (1-m_i)\ell_i + m_i\alpha_p^w$. (Again, we assumed $\alpha_p^w = \alpha_m^w$.) Substituting it and rearranging,

$$\frac{d\ln p_i/p}{d\ln w} = b_i(1-m_i)(\ell_i - \alpha_p^w) \tag{A.14}$$

and

$$\frac{d\ln Y_i/Y}{d\ln w} = \epsilon_i b_i(1-m_i)(\alpha_p^w - \ell_i). \tag{A.15}$$

Now we use equation (A.15) to obtain

$$\begin{aligned}
\frac{1}{k(1-k)}\sum_i (k_i-k)\theta_i \frac{d\ln Y_i}{d\ln w} &= \frac{1}{k(1-k)}\sum_i (k_i-k)\theta_i\epsilon_i b_i(1-m_i)(\alpha_p^w - \ell_i) \\
&= \frac{1}{k(1-k)}\sum_i (k_i-k)\theta_i\bar{\epsilon}_e^w(1-m_i)(\alpha_p^w - \ell_i) \\
&= \frac{1}{k(1-k)}\sum_i (k_i-k)\theta_i\bar{\epsilon}_e^w(1-\bar{m}_e^w)(\alpha_p^w - \ell_i) \\
&= \frac{1}{k(1-k)}\sum_i (k_i-k)\theta_i\bar{\epsilon}_e^w(1-\bar{m}_e^w)\left[\alpha_p^w - (1-k_i)(1-\zeta^w)\right] \\
&= \frac{1}{k(1-k)}\sum_i (k_i-k)\theta_i\bar{\epsilon}_e^w(1-\bar{m}_e^w)(k_i-k)(1-\zeta^w) \\
&= \chi(1-\zeta^w)(1-\bar{m}_e^w)\bar{\epsilon}_e^w.
\end{aligned} \tag{A.16}$$

Note that we denote

$$\bar{\epsilon}_e^w \equiv \frac{\sum_i (k_i-k)(1-m_i)(\alpha_p^w - \ell_i)\theta_i\epsilon_i b_i}{\sum_i (k_i-k)(1-m_i)(\alpha_p^w - \ell_i)\theta_i}.$$

Finally, we obtain α_p from $\sum_i \frac{p_iY_i}{pY}\frac{d\ln p_iY_i/pY}{d\ln w} = 0$.

$$\begin{aligned}
\sum_i \frac{p_iY_i}{pY}\frac{d\ln Y_i/Y}{d\ln w} &= \sum_i \frac{p_iY_i}{pY}(\epsilon_i - 1)b_i(1-m_i)(\alpha_p^w - \ell_i) = 0 \\
\Rightarrow \alpha_p^w &= \frac{\sum_i p_iY_i(\epsilon_i-1)b_i(1-m_i)\ell_i}{\sum_i p_iY_i(\epsilon_i-1)b_i(1-m_i)}
\end{aligned}$$

Finally, from equations (A.8) and (A.16), we now have

$$\bar{\sigma}_e^w = (1-\chi)\sigma_e + \chi\left[\zeta^w \sigma_s + (1-\zeta^w)\bar{m}_e^w \sigma_m + (1-\zeta^w - (1-\zeta^w)\bar{m}_e^w)\bar{\epsilon}_e^w\right],$$

where

$$\zeta^w \equiv \frac{\sum_i (k_i - k)(1-k_i)\theta_i s_i}{\sum_i (k_i - k)(1-k_i)\theta_i},$$
$$\bar{m}_e^w \equiv \frac{\sum_i (k_i - k)(\ell_i - \alpha_p^w)\theta_i m_i}{\sum_i (k_i - k)(\ell_i - \alpha_p^w)\theta_i}$$
$$\bar{\epsilon}_e^w \equiv \frac{\sum_i (k_i - k)(1-m_i)(\alpha_p^w - \ell_i)\theta_i \epsilon_i b_i}{\sum_i (k_i - k)(1-m_i)(\alpha_p^w - \ell_i)\theta_i}$$
$$\alpha_p^w = \frac{\sum_i p_i Y_i(\epsilon_i - 1)b_i(1-m_i)\ell_i}{\sum_i p_i Y_i(\epsilon_i - 1)b_i(1-m_i)}.$$

Software-labor substitution Similar to the proof in the equipment-labor substitution, we begin with

$$\bar{\sigma}_s^w - 1 = \frac{d\ln\frac{s}{1-s}}{(1-k)d\ln w} = \frac{1}{\ell s}\frac{ds}{d\ln w}. \tag{A.17}$$

From the definition, $s \equiv \sum \omega_i s$, we also have

$$ds = \sum_i \omega_i ds_i + \sum_i s_i d\omega_i. \tag{A.18}$$

From Proposition 1, we know that

$$ds_i = (\sigma_s - 1)s_i(1-s_i)(1-k_i)d\ln w = (\sigma_s - 1)s_i \ell_i d\ln w \tag{A.19}$$

Now denote

$$\xi^w \equiv -\frac{\sum_i (\ell_i - \ell)(s_i - s)\omega_i}{\ell s}.$$

Then it is straightforward to check

$$(1-\xi^w) = \frac{\sum_i \ell_i s_i \omega_i}{\ell s},$$

and so $\sum_i \omega_i ds_i = \ell s(1-\xi^w)(\sigma_s - 1)$ from equation (A.19).

Therefore, from equation (A.17) and (A.18), we get

$$\bar{\sigma}_s^w - 1 = (1-\xi^w)(\sigma_s - 1) + \frac{1}{s\ell}\sum_i s_i \frac{d\omega_i}{d\ln w}. \tag{A.20}$$

Since $\omega_i = \frac{(1-m_i)c_i}{(1-m)c}$, following similar steps in deriving equation (A.9) and (A.10), we have

$$\frac{d\ln\omega_i}{d\ln w} = \frac{d\ln(1-m_i)/(1-m)}{d\ln w} + \frac{d\ln c_i/c}{d\ln w}$$

$$= \frac{d\ln(1-m_i)}{d\ln w} + \frac{d\ln Y_i}{d\ln w} + \ell_i(1-m_i) + m_i\alpha_p^w - \frac{d\ln(1-m)}{d\ln w} - \frac{d\ln c}{d\ln w}.$$

Since $\sum_i s_i \frac{d\omega_i}{d\ln w} = \sum_i (s_i - s)\frac{d\omega_i}{d\ln w} = \sum_i (s_i - s)\omega_i \frac{d\ln\omega_i}{d\ln w}$,

$$\begin{aligned}
\frac{1}{s\ell}\sum_i (s_i - s)\omega_i \frac{d\ln\omega_i}{d\ln w} &= \frac{1}{s\ell}\sum_i (s_i - s)\omega_i \left[m_i(\ell_i - \alpha_p^w)(1-\sigma_m) + (1-m_i)(\ell_i - \alpha_p) + \frac{d\ln Y_i}{d\ln w}\right] \\
&= \frac{1}{s\ell}\sum_i (s_i - s)\omega_i \left[\bar{m}_s^w(\ell_i - \alpha_p^w)(1-\sigma_m) + (1-\bar{m}_s^w)(\ell_i - \alpha_p) + \frac{d\ln Y_i}{d\ln w}\right] \\
&= \frac{1}{s\ell}\sum_i (s_i - s)\omega_i \left[\ell_i(1-\bar{m}_s^w\sigma_m) + \frac{d\ln Y_i}{d\ln w}\right] \\
&= \frac{1}{s\ell}\sum_i (s_i - s)\omega_i \left[\ell_i(1-\bar{m}_s^w\sigma_m) + \epsilon_i b_i (1-m_i)(\alpha_p^w - \ell_i)\right] \\
&= \frac{1}{s\ell}\sum_i (s_i - s)\omega_i \left[\ell_i(1-\bar{m}_s^w\sigma_m) + \bar{\epsilon}_s^w (1-\bar{m}_s^w)(\alpha_p^w - \ell_i)\right] \\
&= \frac{1}{s\ell}\sum_i (s_i - s)\omega_i \ell_i \left[(1-\bar{m}_s^w\sigma_m) - \bar{\epsilon}_s^w(1-\bar{m}_s^w)\right] \\
&= \frac{1}{s\ell}\sum_i -(s_i - s)(\ell_i - \ell)\omega_i \left[(\bar{m}_s^w\sigma_m - 1) + \bar{\epsilon}_s^w(1-\bar{m}_s^w)\right] \\
&= \xi^w(\bar{m}_s^w\sigma_m + (1-\bar{m}_s^w)\bar{\epsilon}_s^w - 1).
\end{aligned} \tag{A.21}$$

Here, we denote

$$\bar{m}_s^w \equiv \frac{\sum_i (s_i - s)(\ell_i - \alpha_p^w)\omega_i m_i}{\sum_i (s_i - s)(\ell_i - \alpha_p^w)\omega_i}, \text{ and } \bar{\epsilon}_s^w \equiv \frac{\sum_i (s_i - s)(1-m_i)(\alpha_p^w - \ell_i)\omega_i\epsilon_i b_i}{\sum_i (s_i - s)(1-m_i)(\alpha_p^w - \ell_i)\omega_i}$$

Inserting equation (A.21) into equation (A.20), we get

$$\bar{\sigma}_s^w = (1-\xi^w)\sigma_s + \xi^w\left[\bar{m}_s^w\sigma_m + (1-\bar{m}_s^w)\bar{\epsilon}_s^w\right], \tag{A.22}$$

where

$$\begin{aligned}
\xi^w &= -\frac{\sum_i (s_i - s)(\ell_i - \ell)\omega_i}{s\ell} \\
\bar{m}_s^w &= \frac{\sum_i (s_i - s)(\ell_i - \alpha_p^w)\omega_i m_i}{\sum_i (s_i - s)(\ell_i - \alpha_p^w)\omega_i} \\
\bar{\epsilon}_s^w &= \frac{\sum_i (s_i - s)(1-m_i)(\alpha_p^w - \ell_i)\omega_i\epsilon_i b_i}{\sum_i (s_i - s)(1-m_i)(\alpha_p^w - \ell_i)\omega_i}
\end{aligned}$$

Cases with $d\ln r > 0$ and $d\ln q > 0$ are analogous. ∎

Proof of Proposition 3 Consider a case with $d\ln w > 0$, $d\ln r = 0$, and $d\ln w = 0$.

We define aggregate markup as $\mu := \sum_i \mu_i\omega_i$. Note that we weight the establishment i's markup with i's share of non-material cost ($\omega_i = (wL_i + rK_i + qS_i)/(wL + rK + qS)$).

Changes in the aggregate markup responding to a change in wage is given by

$$\frac{d\ln\mu}{d\ln w} = \frac{1}{\mu}\frac{d\mu}{d\ln w} = \frac{1}{\mu}\left[\sum_i \omega_i \frac{d\mu_i}{d\ln w} + \sum_i \mu_i \frac{d\omega_i}{d\ln w}\right] \tag{A.23}$$

Since $p_i = \mu_i \times mc_i$, from the definition of price pass-through ($b_i \equiv \frac{d\ln p_i}{d\ln mc_i}$) and Shephard's lemma ($\frac{d\ln mc_i}{d\ln w} = \ell_i(1-m_i) + m_i\alpha_p^w$), we have

$$\frac{d\ln\mu_i}{d\ln w} = (b_i - 1)[\ell_i(1-m_i) + m_i\alpha_p^w] = (b_i-1)\tilde{\ell}_i, \tag{A.24}$$

where we denote $\tilde{\ell}_i \equiv \ell_i(1-m_i) + m_i\alpha_p^w$ for notational convenience. Using equation (A.24),

$$\sum_i \omega_i \frac{d\mu_i}{d\ln w} = \sum_i \omega_i\mu_i\tilde{\ell}_i(b_i-1) = \mu\tilde{\ell}(1-\eta^w)(\bar{b}^w - 1), \tag{A.25}$$

where, we define

$$\eta^w := -\frac{\sum_i(\mu_i-\mu)(\tilde{\ell}_i-\tilde{\ell})\omega_i}{\mu\tilde{\ell}}, \text{ and } \bar{b}^w := \frac{\sum_i \mu_i\tilde{\ell}_i b_i\omega_i}{\sum_i \mu_i\tilde{\ell}_i\omega_i}.$$

Also, following a similar calculation in equation (A.21),

$$\begin{aligned}
\sum_i(\mu_i-\mu)\omega_i\frac{d\ln\omega_i}{d\ln w} &= \sum_i(\mu_i-\mu)\omega_i\left[m_i(\ell_i-\alpha_p^w)(1-\sigma_m) + \tilde{\ell}_i + \frac{d\ln Y_i}{d\ln w}\right] \\
&= \sum_i(\mu_i-\mu)\omega_i\left[(\tilde{\ell}_i-\alpha_p^w)(\sigma_m-1) - (\ell_i-\alpha_p^w)(\sigma_m-1) + \tilde{\ell}_i - \epsilon_i b_i(\tilde{\ell}_i-\alpha_p^w)\right] \\
&= \sum_i(\mu_i-\mu)\omega_i\left[-\ell_i(\sigma_m-1) + \tilde{\ell}_i\sigma_m - \epsilon_i b_i(\tilde{\ell}_i-\alpha_p^w)\right] \\
&= \sum_i(\mu_i-\mu)\omega_i\left[-\ell_i(\sigma_m-1) + \tilde{\ell}_i(\sigma_m-\bar{\epsilon}_\mu^w)\right] \\
&= \sum_i(\mu_i-\mu)\omega_i(\ell_i-\ell)(1-\sigma_m) + \sum_i(\mu_i-\mu)\omega_i(\tilde{\ell}_i-\tilde{\ell})(\sigma_m-\bar{\epsilon}_\mu^w) \\
&= \mu\tilde{\ell}\iota^w(\sigma_m-1) + \mu\tilde{\ell}\eta^w(\bar{\epsilon}_\mu^w-\sigma_m),
\end{aligned} \tag{A.26}$$

where, we define

$$\bar{\epsilon}_\mu^w := \frac{\sum_i(\mu_i-\mu)(\tilde{\ell}_i-\alpha_p^w)\omega_i\epsilon_i b_i}{\sum_i(\mu_i-\mu)(\tilde{\ell}_i-\alpha_p^w)\omega_i}, \text{ and } \iota^w := -\frac{\sum_i(\mu_i-\mu)(\ell_i-\ell)\omega_i}{\mu\tilde{\ell}}.$$

Combining equation (A.23), (A.25), and (A.26), we get

$$\frac{d\ln\mu}{\tilde{\ell}d\ln w} = (1-\eta^w)\bar{b}^w + \eta^w\bar{\epsilon}_\mu^w - 1 + (\iota^w-\eta^w)(\sigma_m-1),$$

with

$$\eta^w \equiv -\frac{\sum_i(\mu_i-\mu)(\tilde{\ell}_i-\tilde{\ell})\omega_i}{\mu\tilde{\ell}}$$

$$\bar{b}^w \equiv \frac{\sum_i \mu_i \tilde{\ell}_i b_i \omega_i}{\sum_i \mu_i \tilde{\ell}_i \omega_i}$$

$$\bar{\epsilon}^w_\mu \equiv \frac{\sum_i (\mu_i - \mu)(\tilde{\ell}_i - \alpha^w_p)\omega_i \epsilon_i b_i}{\sum_i (\mu_i - \mu)(\tilde{\ell}_i - \alpha^w_p)\omega_i}$$

$$\iota^w \equiv -\frac{\sum_i (\mu_i - \mu)(\ell_i - \ell)\omega_i}{\mu \tilde{\ell}}$$

Cases with $d \ln r > 0$ and $d \ln q > 0$ are analogous. ∎

Proof of Proposition 4 Denoting firm i's capital other than equipment and software with O_i and its price o, the aggregate labor income share is

$$\begin{aligned} LS &= \frac{\sum_i wL_i}{\sum_i p_i Y_i} = \frac{\sum_i wL_i}{\sum_i \mu_i (wL_i + rK_i + qS_i) + \sum_i \mu_i o O_i} \\ &= \frac{\sum_i \ell_i \omega_i}{\sum_i \mu_i (\omega_i + \frac{oO_i}{wL_i + rK_i + qS_i}\omega_i)} \\ &= \frac{\ell}{\sum_i \mu_i \omega_i (1 + o_i)}. \end{aligned}$$

For $\sum_i \mu_i \omega_i (1 + o_i)$,

$$d \sum_i \mu_i \omega_i (1 + o_i) = \sum_i d(\mu_i \omega_i)(1 + o_i) + \sum_i \mu_i \omega_i d(1 + o_i).$$

Assuming, $\sigma_o = 1$, we have $\sum_i \mu_i \omega_i d(1 + o_i) = 0$, and hence

$$d \sum_i \mu_i \omega_i (1 + o_i) = \sum_i d(\mu_i \omega_i)(1 + o_i).$$

That is, in principle, the distribution of the income share of other capital across firms can affect the weights for aggregating markups. To simplify the decomposition, however, we assume $(1 + o_i) = (1 + \bar{o})$ for all i. Then we have

$$\frac{dLS}{LS} = d \ln \ell - d \ln \mu.$$

Therefore, the impact of software-embodied technological change on aggregate labor share is given by

$$\frac{dLS}{-d \ln q} = -LS \times \left[s(\bar{\sigma}^q_s - 1) - \tilde{s}(\bar{\sigma}^q_\mu - 1) \right].$$

Similarly, the impact of equipment-embodied technological change is

$$\frac{dLS}{-d \ln r} = -LS \times \left[-se(\bar{\sigma}^r_s - 1) + e(\bar{\sigma}^r_e - 1) - \tilde{\kappa}(\bar{\sigma}^r_\mu - 1) \right].$$

Finally, an increase in wages results in

$$\frac{dLS}{d \ln w} = -LS \times \left[s(1 - e)(\bar{\sigma}^w_s - 1) - (1 - e)(\bar{\sigma}^w_e - 1) + \tilde{\ell}(\bar{\sigma}^w_\mu - 1) \right].$$

∎

Appendix B Data

B.1 National Accounts

B.1.1 Price index

When measuring capital-embodied technological change, we rely on the price indices of investment and consumption goods obtained from the Korean national accounts. A notable drawback of using the Korean national accounts is that the software price index would not fully capture software-embodied technological change. Since 1994, the software price index comes from the producer price index, potentially inadequately reflecting quality improvements. To address this concern, we adopt the adjustment method proposed by Parker and Grimm (2000), the approach taken by the US Bureau of Economic Analysis (BEA). Specifically, BEA makes a bias adjustment of 3.15 percent per year to the producer price index from the Bureau of Labor Statistics due to the discrepancy between the hedonic method and the matched model method. Accordingly, we apply a bias adjustment of 3.15 percent per year to the software price index from 1994 onward. The resulting software price index in Korea exhibits similarity with the BEA price index for the US, affirming the reliability of our method, given that software technology is expected to be consistent between Korea and the US.

B.1.2 The Rate of Return on Capital

A standard macro model predicts that the rate of return on capital for each capital type j satisfies the following condition:

$$R_{j,t} = (1 + r_t)q_{j,t} - (1 - \delta_{j,t})q_{j,t-1},$$

where r_t is the net rate of return, $q_{j,t}$ is the price of capital relative to consumption for capital type j, and $\delta_{j,t}$ is the depreciation rate for capital type j. We calculate $q_{j,t}$ using the capital price index and the price of consumption in the National Accounts (with a bias-adjusted software price, as described earlier). The depreciation rate $\delta_{j,t}$ is computed from net capital stock and investment using the formula:

$$\delta_{j,t} = \frac{NK_{j,t-1} \times \pi^k_{j,t} - NK_{j,t} + NI_{j,t}}{NK_{j,t-1} \times \pi^k_{j,t}},$$

where $NK_{j,t}$ is the nominal capital stock at the end of period t for capital type j, $NI_{j,t}$ is the nominal investment at time t for capital type j, and $\pi^k_{j,t}$ is the ratio of capital price index between time t and $t-1$ for capital type j.

For the net rate of return, r_t, we consider two approaches. First we estimate r_t by the corporate bond rate net of expected inflation, where expected inflation is measured by a three-year moving average of the CPI inflation rate. Second, we use the estimated aggregate markup and equation (B.1) to back out r_t:

$$LS_t = \frac{wL_t}{\mu_t(wL_t + \sum_j R_{j,t}K_{j,t})}, \tag{B.1}$$

where wL_t is the compensation of employees (adjusted for proprietor's income) and μ_t is the aggregate markup estimated from the firm-level data. We chose the second approach in our decomposition analysis. However, the specific choice of approach makes a minimal difference in our results.

B.1.3 Labor Share

Total income in the national economy includes the proprietors' income, a mix of labor and capital income. Gollin (2002) suggested several methods to estimate the labor share of the proprietors' income. One such method assumes that the labor share in proprietor's income is the same as that of the rest of the economy (M1):

$$\text{M1} : LS = \frac{CE + NLS * PI}{Y},$$
$$\text{with } NLS = \frac{CE}{Y - CFC - PI}$$

where CE represents compensation of employees, PI is proprietors' income, CFC is consumption of fixed capital, NLS is net labor share, and Y is either gross value added (without net production tax) or gross domestic product (with net production tax). To be consistent with the theory in this paper, we compute gross labor share, which includes the consumption of fixed capital in the denominator.

Given the substantial presence of self-employment in the Korean economy, the measurement of labor share is sensitive to the treatment of the proprietor's income. In particular, Park (2020) highlighted that a sizable proportion of self-employed is working with employees, whose wages are included in the compensation of employees measured in the national accounts. He proposed explicitly considering this factor in constructing aggregate labor share. Specifically, he utilizes information that the ratio of business income between self-employed with employees and without employee is approximately 2.3, and self-employed with employees hire 2.5 workers on average. Assuming that labor share of self-employed without employees is the same as labor share of the rest of the economy, we get the following (M2):

$$\text{M2} : LS = \frac{CE + NLS * PI_0 + NLS * PI_e/3.5}{Y},$$

$$\text{with } PI_0 = PI \times \frac{\text{self}_0}{\text{self}_0 + 2.3 \times \text{self}_e}$$
$$PI_e = PI - PI_0$$
$$NLS = \frac{CE}{Y - CFC - PI_0}$$

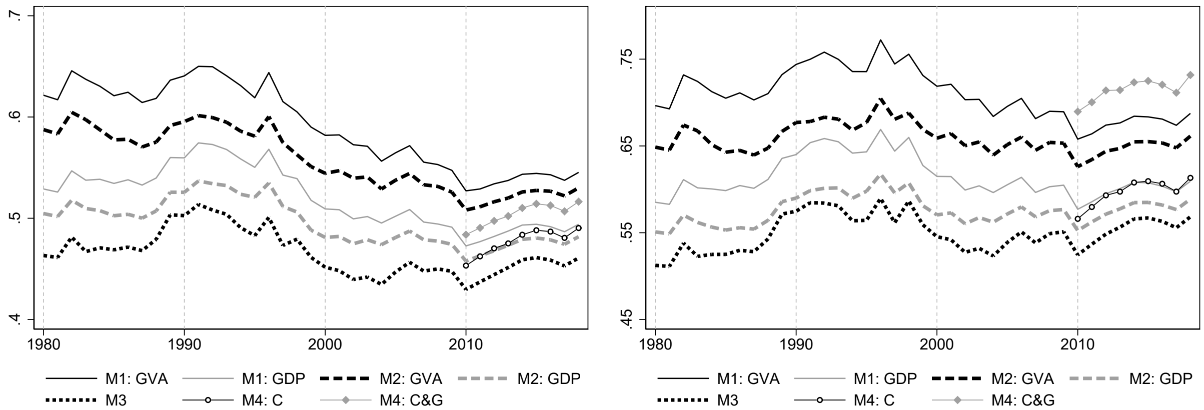
where self_e and self_0 are the number of self-employed with and without employees, respectively. Also, PI_e and PI_0 refer to the income of self-employed with and without employees, respectively.

Alternatively, one can assume that labor share of the entire self-employed is the same as the labor share of the corporate sector (M4: C) or the rest of the economy (M4: C&G). To do so, we need to subtract labor income of workers working with self-employed from the compensation of employees. Unfortunately, Korean national accounts provide such information only from 2010. Instead, we can assume that the wage of workers working with self-employed is the same as the average wage of workers working in an establishment with the size of 5 to 9 employees, with an average number of workers working with self-employed being 3.5 (M3):

$$\text{M3} : LS = \frac{CE \times (1 - \frac{\text{self}_0 \times 2.5}{\text{wemp}} \times wr) + NLS * PI}{Y},$$
$$\text{with } NLS = \frac{CE \times (1 - \frac{\text{self}_0 \times 2.5}{\text{wemp}} \times wr)}{Y - CFC - PI}$$

where wemp is wage employment and wr is the ratio of the average wage of workers working in an establishment with size of 5 to 9 relative to the average wage of workers in all establishments.

Figure B.1 shows the calculated aggregate labor share using alternative approaches. While the magnitude of the decrease in labor share varies depending on methods, the patterns are generally similar, especially for gross labor share. For net labor share, the declining trend is mitigated when adjusting for self-employment with employees (M2). Because the rate of return on capital should include depreciation, we stick to gross labor share in our analysis. This is particularly crucial considering our focus on software, which typically has a higher depreciation rate than other factors. It is noteworthy that Karabarbounis and Neiman (2014) showed in a simple model that during a transition where the main shock to the economy is a decline in the price of high depreciation capital, gross labor share falls while net labor share rises. Importantly, in such cases, gross labor share serves as a better measure of changes in welfare among agents. In the main text, we use a conservative measure, M2:GDP, as a baseline measure of aggregate labor share.



(a) Gross Labor Share (b) Net Labor Share

Fig. B.1: Aggregate labor share in Korea

M1 assumes that labor share in income of self-employed is the same as that of the rest of the economy. M2 assumes that labor share in income of self-employed without employees is the same as that of the rest of the economy. M3 assumes that the average wage of employees working with self-employed is equal to the average wage of workers working in plants with size 5 to 9. M4 calculates labor share of the corporate sector (M4-C) or corporate and government (M4-C&G). The denominator is either GDP or GVA in gross labor share and either GDP or GVA net of consumption of fixed capital in net labor share.

B.2 KISDATA

KISDATA is a database on financial information for firms listed on the Korea Stock Exchange and firms unlisted but required to publish external auditing reports. The criteria for external audit requirement is as follows. Until 2008, firms whose asset value exceeded 7 billion KRW had to be audited externally. Since 2009 (2014), firms with (i) asset value greater than 10 billion (12 billion) KRW, (ii) asset value greater than 7 billion KRW and liability greater than 7 billion KRW, or (iii) asset value greater than 7 billion KRW and more than 300 employees were subject to external audits. Among firms in KISDATA, we exclude financial firms and quasi-governmental and non-profit firms from the sample. Our data runs from 2003 to 2018.

B.2.1 Labor Share

To construct labor shares, we need data on labor compensation and value added. We combine employee compensation and benefits in income statements and labor costs in manufacturing cost statements to obtain firms' total labor compensation. Note that the employee compensation and benefits in an income statement can be understood as the labor income accruing to non-production workers, while the labor cost in a manufacturing cost statement as the labor income accruing to production workers.

To compute value added, we add up operating profit, depreciation and amortization, taxes and dues, and labor compensation. The labor share at the firm level is then computed by dividing labor compensation by value added. In the regression analysis, we restrict our sample to those with labor shares between zero and one.

B.2.2 Software and Equipment Capital

We use the variable "intangible asset - software" in firms' balance sheets to measure their software asset. According to Korean Generally Accepted Accounting Principles (K-GAAP), a firm classifies its software purchases from outside as software assets. A firm may have software developed in-house as an intangible asset, but this component is not included in our analysis. It can be included in research and development in principle, but not separately reported. Our measure of equipment capital is the sum of machinery, transportation equipment, tools, fixtures, and furniture, reported in balance sheets. For the reduced form regression in Section 2, we divide software asset and equipment asset by value added to measure software intensity and equipment intensity, respectively.

B.2.3 Markup

We consider five different approaches to measure firm-level markups. Our procedure for the construction of markup follows Baqaee and Farhi (2019) and Edmond et al. (2018) to a large extent.

Accounting Profit (AP) For the accounting profit approach, we use operating profit to measure profits and use the expression

$$\text{operating profit} = \left(1 - \frac{1}{\mu^{AP}}\right) \text{sales} \tag{B.2}$$

to get μ^{AP} for each firm in each year.

User Cost (UC) We assume that the operating surplus is

$$OS = RK + \left(1 - \frac{1}{\mu^{UC}}\right) sales, \tag{B.3}$$

where OS is the operating income (with depreciation), R is the user cost of capital, and K is the stock of capital. We compute the sum of sales net of cost of goods sold and depreciation as OS, and the sum of tangible and intangible assets as K.

The user cost of capital is given by

$$R_{i,t} = (1 + r_t) - (1 - \delta_{i,t}) E_t p^k_{i,t+1} / p^k_{i,t}, \tag{B.4}$$

where r_t is the average real rate of corporate bond, $\delta_{i,t}$ is the industry-level depreciation rate implied in the National Account, and $E_t p^k_{i,t+1}/p^k_{i,t}$ is the three-year moving average of the changes in the relative price of capital to consumption by industry. We then back out μ^{UC} from equations (B.3) and (B.4).

Production Function (PF) For the production function approach, we estimate the elasticity of output with respect to variable inputs following Baqaee and Farhi (2019) and De Loecker et al. (2020).

To estimate elasticity, we need an outcome variable (log sales), free variable (log cost of goods sold), state variable (log capital stock), and proxy variable (log investment). We deflate sales and cost of goods sold with gross value added deflator by industry and deflate capital expenditure with gross fixed capital formation deflator by industry. To compute capital stock, we apply the perpetual inventory method (PIM) with the initial level of tangible and intangible capital and capital expenditure. We also control for sales shares in one-digit and two-digit industries in the estimation. We exclude samples with the cost of goods sold to sales ratio or selling, the general and administrative expense to sales ratio in the top and bottom 2.5 percentiles by year. We also exclude agriculture and the finance and insurance industry.

The elasticity is estimated as in Olley and Pakes (1996) with three-year rolling windows by one-digit industry. Then μ^{PF} is

$$\mu^{PF} = \frac{\partial \log F / \partial \log X}{X/Y}, \tag{B.5}$$

where F is the production function, X is variable input (cost of goods sold), and Y is sales turnover.

Cost Minimization (CM1 & CM2) One of main challenges in estimating markup lies in the difficulty of estimating the output elasticity ($\partial \log F / \partial \log X$) in equation (B.5). For instance, Bond et al. (2021) argues that production function estimation based on revenue data (as in PF) offers little insight into firm-level markups, while Ridder et al. (2022) proposes that revenue data can still offer valuable information on the dispersions and trends of markups, albeit with a biased estimation of markup levels.

One way to circumvent this issue is to indirectly use an optimal condition derived from cost minimization. Considering labor as a variable input, for example, cost minimization under the constant returns to scale technology implies:

$$\frac{\partial \log F_i}{\partial \log \ell_i} = \frac{w\ell_i}{w\ell_i + rK_i + qS_i + pM_i} \tag{B.6}$$

Instead of directly estimating the output elasticity, one can use equation (B.6) together with equation (B.5) to compute the markup. In practice, one can use either firm-level cost share (CM1, Oberfield and Raval, 2021) or cost-weighted average at industry-level (CM2, Edmond et al., 2018).

Selection of Benchmark Markup When computing distributional moments according to Proposition 2 and 3, we choose the markup estimated according to cost minimization (CM1) as the benchmark, and compute the demand elasticity that each firm faces according to:

$$\epsilon_i = \frac{\mu_i}{\mu_i - 1} \tag{B.7}$$

Because we need $\mu_i > 1$ to apply equation (B.7), we trimmed observations with a markup less than 1.01 as well as those greater than 10. We selected markup CM1 as the benchmark for mainly four reasons summarized in Table B.1. First, among the considered markup measures, CM1 has the fewest observations with $\hat{\mu} < 1$ that are trimmed in the analysis. Second, the markup CM1 has the fewest missing values for observations with positive software assets, which are crucial in our analysis. Third, CM1 is a markup measure that aligns directly with our theory. Fourth, all the markup measures produce similar results in the quantification, except for the case of CM2, which shows about twice the magnitude of effects on aggregate markup.

	Share of obs. w/		# missing μ with sft>0	Decomposition results		
	$\mu < 1.01$	$\mu > 10$		Overall	Substitution	Markup
AP	0.231	0.000	8,352	-0.029	-0.020	-0.008
UC	0.089	0.000	8,701	-0.027	-0.015	-0.012
PF	0.330	0.005	16,435	-0.027	-0.015	-0.012
MC1	0.058	0.006	1,766	-0.029	-0.014	-0.015
MC2	0.196	0.027	9,425	-0.051	-0.014	-0.038

Table B.1: Summary on alternative markups

The first and second columns show the share of observations with estimated markup below 1.01 and above 10, respectively. The third column counts the number of missing or trimmed observations with information on software assets. The fourth to sixth columns show decomposition results on the effects of software-embodied technological change, as in Section 5, with alternative markup measures.

B.2.4 Pass Through

We compute each firm's price pass-through (b_i) using the relationship between the markup and the pass-through under the demand aggregator proposed by Klenow and Willis (2016), which is represented by equation (38), or

$$b_i = \frac{1}{1 - \frac{\mu' x}{\mu}} = \frac{1}{1 + \mu_i \frac{\nu}{\sigma}}.$$

We follow Edmond et al. (2018) when setting a value to super-elasticity, or ν/σ. Specifically, under the Klenow-Willis specification, each firm's sales share and markup follow the relationship:

$$\frac{1}{\mu_{i,t}} + \ln\left(1 - \frac{1}{\mu_{i,t}}\right) = \text{constant} + \frac{\nu}{\sigma} \ln sh_{i,t}, \tag{B.8}$$

where $sh_{i,t}$ represents the sales share of firm i at time t. We estimate equation B.8 controlling for firm and year fixed effects. The estimated parameter for the super-elasticity ν/σ is 0.113, with a standard error clustered at the firm-level of 0.002[27].

B.3 Census

Our data source for the cross-sectional estimation of the elasticity of substitution between labor and capital is the Korean Economic Census 2015. It surveys all establishments in the manufacturing sector with more than one employee as of December 31, 2015. We exclude branches, sole proprietorships, governmental and non-profit establishments as they do not report intangible assets. We use annual payroll for wL_i, equipment capital (machinery and transportation equipment) for K_i, and software assets for S_i. We drop all establishments that did not report whether they have intangibles. When an establishment explicitly reports that it does not hold intangibles, we assign zero to its S_i. To compute the factor income shares, we use the rate of return on equipment and software (r and q) imputed from the National Accounts. We winsorize factor shares at the first and the 99th percentiles.

B.3.1 Distribution of Factor Shares

The Census data includes the location of establishments. The unit of a region in our analysis is Si-Gun-Gu, an administrative division of Korea, comparable with commuting zones in the US in terms of the average population size. Figure B.2 shows the regional distribution of software (s_i), equipment (k_i), and labor shares (ℓ_i) in non-material costs.

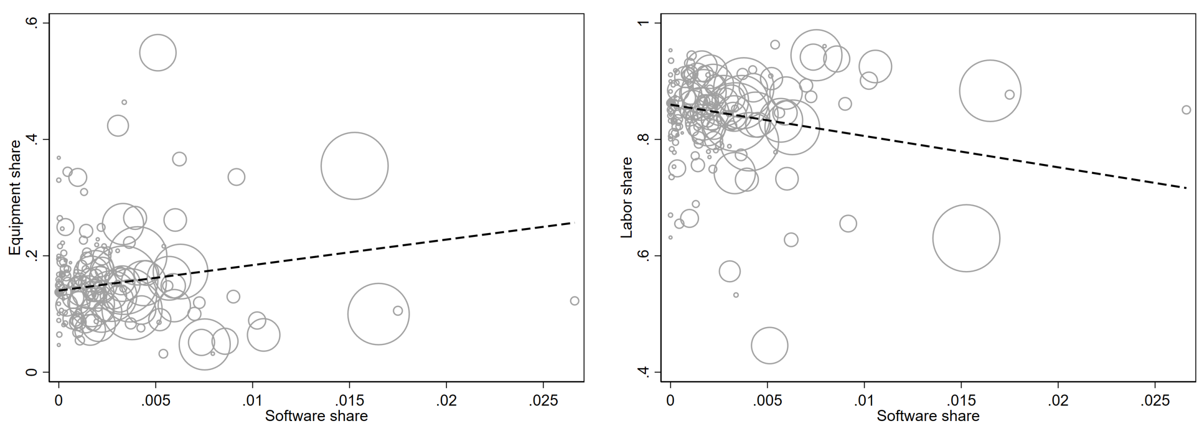
[27] Edmond et al. (2018) estimated the same equation using US data and obtained a super-elasticity of 0.16.

(a) Software (s_i) (b) Equipment (k_i) (c) Labor (ℓ_i)

Fig. B.2: Factor Expenditure Shares by Region

The colors represent the quantiles of regions in the distribution of factor shares on non-material cost, with darker colors corresponding to higher quantiles.

Although not visually apparent, the covariance between software shares (s_i) and equipment shares (k_i) is positive and the covariance between software shares (s_i) and labor shares (ℓ_i) is negative, indicating that factor shares are not always negatively correlated in the case of three factors. These relationships are more clear in Figure B.3.



(a) Software Share vs. Equipment Share (b) Software Share vs. Labor Share

Fig. B.3: Relationship between Software Share and Labor or Equipment Share

Each circle is a region. The horizontal axis is the average software share of all firms in a region. The vertical axis is a region's equipment share in the left panel and its labor share in the right panel. The size of the circle corresponds to a region's value added.

B.4 Regional Employment Survey

We use wage differences across regions as our explanatory variable when estimating the micro elasticities of substitution. We obtain the wages from the 2015 Regional Employment Survey. The Regional Employment Survey is a household-level survey of workers' salary, demographic information, educational attainment, and experience. We control for workers' observable characteristics such as education, experience, and demographics. We then aggregate the residual wages to the region level for regional variation in labor costs.

B.4.1 Bartik instrument

The residualized local wages may be correlated with unobservable productivity that is not factor-neutral. To address this concern, we use Bartik (1991) instrument. Our instrument is

$$Z_r = \sum_{i \in N_s} \omega_{r,i,0} \log(L_{i,t}/L_{i,0})$$

where N_s is the set of service industries, $\omega_{r,i,0}$ is the industry i's share of employment in region r at time 0, and $L_{i,t}$ is the nationwide employment of industry i at time t.

According to Goldsmith-Pinkham et al. (2020), the Bartik estimator $\hat{\beta}_{bartik}$ can be rewritten as the weighted sum of the just-identified estimators with one industry's regional share as an instrument, $\hat{\beta}_{bartik} = \sum_i \hat{\alpha}_i \hat{\beta}_i$, where α_i's are known as Rotemberg weights. Table B.2 is a summary of the Rotemberg weights. Panels A and C show that two industries with the largest Rotemberg weights account for 95 percent (=0.527+.425) of the overall weights and 65 percent (=0.952/1.457) of the positive weights in the estimator. They are the research and development industry and business support services industry. The contribution of these two industries are shown by the two large circles in Figure B.4.

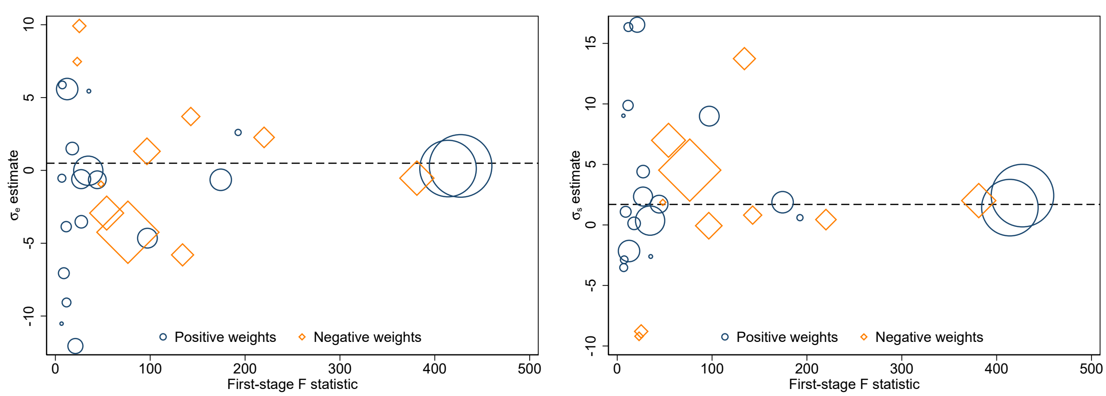
Panel B shows that the national growth rates of service industries indexed by i ($g_i = L_{i,t}/L_{i,0}$) are also positively correlated with the weights, while the variance of an industry share across regions ($var(\omega_i)$) are only weakly correlated with the weights.

Regarding the identification with Bartik instruments, Borusyak et al. (2021) shows that one needs to assume either the initial shares are exogenous or the shocks are exogenous. In particular, a Bartik regression can be translated into a shock-level regression with a specific weight ($s_i = \sum_r \omega_{r,i}$) when shocks are exogenous. For an original IV regression $y_r = \alpha + \beta x_r + \varepsilon_r$ with Bartik instrument z_r, an equivalent shock-level

	Sum	Mean	Share		
Panel A. Negative and positive weights					
Negative	-0.457	-0.038	0.239		
Positive	1.457	0.066	0.761		
	$\hat{\alpha}_i$	g_i	$\hat{F}_i$	$var(\omega_i)$	
Panel B. Correlations					
$\hat{\alpha}_i$	1.000				
g_i	0.244	1.000			
$\hat{F}_i$	0.597	0.114	1.000		
$var(\omega_i)$	-0.034	-0.104	0.098	1.000	
	$\hat{\alpha}_i$	g_i	$\hat{\sigma}_e$	$\hat{\sigma}_s$	$\sum_{k=1}^{i} \hat{\alpha}_k$
Panel C. Top Rotemberg weight industries					
Research & development	0.527	0.316	0.326	2.387	0.527
Business support services	0.425	0.182	0.130	1.427	0.952
Warehousing and transportation	0.108	0.165	0.045	0.319	1.060
Retail trade (except motor)	0.061	-0.023	-0.647	1.904	1.122

Table B.2: Summary of the Rotemberg Weights

Panel B reports correlations among the Rotemberg weights ($\hat{\alpha}_i$), the nationwide employment growth of industry i ($g_i = L_{i,t}/L_{i,0}$), the first-stage F-statistic of the industry share ($\hat{F}_i$), and the variance of a given industry's employment share across regions ($var(\omega_i)$).



(a) Equipment ($\hat{\sigma}_e$)

(b) Software ($\hat{\sigma}_s$)

Fig. B.4: Heterogeneity of the elasticities

The figure plots the relationship between each instruments' $\hat{\sigma}_j$, first-stage F-statistics, and the Rotemberg weights. Each point is a separate instrument's estimate. The estimated ($\hat{\sigma}_j$) for each instrument is on the y-axis and the estimated first-stage F-statistic is on the x-axis. The size of the points is scaled by the absolute value of the Rotemberg weights, with circles for positive weights and diamonds for negative weights. The dashed line is at the estimated elasticity using the Bartik instrument. Instruments with first-stage F-statistics below 5 are excluded.

regression is a s_i-weighted IV regression, using the shocks g_i as instrument, estimating

$$\bar{y}_i = \alpha + \beta \bar{x}_i + \bar{\varepsilon}_i.$$

While the point estimates $\hat{\beta}$ are equivalent in both approaches, Borusyak et al. (2021) shows that the shock-level regression provides a correct inference when shocks are exogenous but shares are endogenous. That is, the Bartik instrument is valid when shocks are randomly assigned (and have a common mean). Regarding the consistency of the estimator, the effective sample size of the shock-level regression is large when the expected Herfindahl index of average shock exposure ($\sum_i s_i^2$) converges to zero as the number of regions increases.

For the shock-level regression, one needs to think carefully about the regional weights $\omega_{r,i}$'s. When $\sum_i \omega_{r,i}$ varies across regions, it can generate non-random variations in addition to quasi-random variations of shocks. Borusyak et al. (2021) shows that one can address this concern by controlling for $\sum_i \omega_{r,i}$ in the regression. Because we construct Bartik instrument using services industries, we can consider two kinds of weights: (i) construct $\omega_{r,i}$ with services employment only, so that $\sum_i \omega_{r,i} = 1$ for all r, or (ii) construct $\omega_{r,i}$ with total employment in the region, so that $\sum_i \omega_{r,i} \leq 1$ varies across r. We consider the former (i) as a benchmark and (ii) as a robustness check.

Table B.3 reports summary statistics of shocks $g_i = L_{i,t}/L_{i,0}$ and the inverse of the Herfindahl index of industry weights ($1/\sum_i s_i^2$). In the regional employment survey, only two-digit industries are reported, so we have a total of 34 service industries. When we construct $\omega_{r,i}$ with total regional employment, a residual industry with share $\omega_{r,0} = 1 - \sum_{i \in N_s} \omega_{r,i}$ can be thought as one with zero growth. Summary statistics in this case are reported in the first column of Table B.3. In the second case, we consider only service industries so that $\sum_{i \in N_s} \omega_{r,i} = 1$, reported in the second column of Table B.3. The non-service industry has a large share of regional employment, and hence it generates a much smaller interquartile range and larger HHI, leading to a smaller effective sample size (Column (1) in Table B.3). This suggests that we want to choose between constructing weights only with service industries (Column (2) in Table B.3).

Table B.4 reports the results from the shock-level regression with only service employment (column 1) and with total employment controlling for the initial share of non-service employment (column 2). We have $\sigma_e < 1$ and $\sigma_s > 1$ in both cases, but the standard errors are much larger than those clustered at the level of region and three-digit industry. These results are also reported in Table 2 as SSIV1 and SSIV2. In column 3 and column 4, we also report shock-level regression results with an instru-

	With Total Employment (1)	With Service Employment (2)
Mean g_i	0.030	0.053
Standard Deviation g_i	0.091	0.116
Interquartile Range g_i	0.005	0.100
$1/HHI$	4.78	12.70
Maximum s_i	0.429	0.155
No. of industries	35	34

Table B.3: Summary Statistics of Shocks (g_i) and Industry Weights (s_i)

ment of which share includes industrial network structure.

The standard errors are large as we have a small effective sample size, and do not utilize establishment level information here. To be specific, as shown in Table B.3, we only have 34 two-digit service industries in the regional employment survey where the inverse of HHI is only 12.70 even when we exclude non-service industries.

	Cross-Sectional		Shock-level	
	Complete share	Incomplete share	Complete share	Incomplete share
Equipment (σ_e)	0.600	-0.164	0.600	-0.164
	(0.153)	(0.165)	(1.051)	(0.815)
Software (σ_s)	1.620	2.736	1.620	2.736
	(0.230)	(0.253)	(0.879)	(0.928)
First stage F	693.7	567.5	7.025	7.201
Obs.	31,403	31,403	34	34

Table B.4: Estimation Results: Shock-level Regression

Appendix C Robustness of Decomposition

In this section, we redo decomposition analysis in Section 5 with alternative approaches in various perspectives.

C.1 Alternative Data

We check whether our decomposition results remain robust with aggregate elasticities computed with Census data. Because we have software information only in the year 2015 for the manufacturing sector, we calculate distributional moments with 2015

Census data for manufacturing plants. Since the distributional moments do not exhibit a clear time trend (Figure 5), our main results will likely remain even if we used the aggregate elasticities based on the 2015 Census distributional moments for all periods.

Table C.1 reports the decomposition with aggregate elasticities computed from Census data. With aggregate elasticities from Census data, the aggregate markup rises less and there is slightly more factor substitution through between-firm reallocation. In addition, equipment-embodied technological change reduces the labor share less due to attenuated between adjustments in factor substitution. Overall, capital-embodied technological changes (software and equipment combined) account for about 57.7 percent of the decline in the aggregate labor share in the data.

- Total changes in labor share : -0.044

- Effects of Software-embodied Technological Change ($\Delta \ln 1/q$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.025	-0.016	-0.015	-0.001	-0.010	-0.007	-0.003
(% explained)	(57.5)	(35.4)	(33.0)	(2.4)	(22.0)	(15.7)	(6.4)

- Effects of Equipment-embodied Technological Change ($\Delta \ln 1/r$)

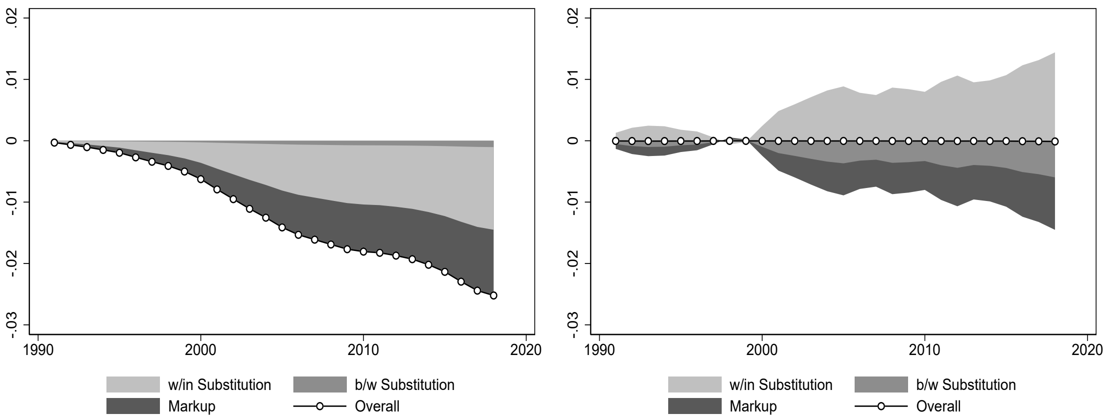
	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.000	+0.008	+0.014	-0.006	-0.009	-0.009	+0.000
(% explained)	(0.3)	(-19.2)	(-32.8)	(13.6)	(19.5)	(19.7)	(-0.2)

Table C.1: Effects of Capital-Embodied Technological Change with Census Data

The decomposition is for the periods between 1990 and 2018. Percent explained of the overall labor share decline in the data in parentheses.

C.2 Alternative Pass-through

Baqaee et al. (2023) argued that the Klenow-Willis specification may produce too little variation in price pass-through, compared to the empirical estimates of the price pass-through from Belgian firm data, potentially attenuating the magnitude of reallocation. To be specific, when estimating b_i from reasonably estimated μ_i using the Klenow-Willis specification, the resulting pass-throughs (b_i) show insufficient variation compared to the empirical b_i provided by Amiti et al. (2019). Instead of relying on



(a) Software Contribution

(b) Equipment. Contribution

Fig. C.1: Effects of Capital-Embodied Technological Change with Census Data

The overall effect is calculated using the macro elasticity ($\bar{\sigma}$) and the within effect is computed based on the micro elasticity (σ) and the average pass-through ($\bar{b}$). The between effect is the difference between the effects based on macro elasticity and micro elasticity.

the Klenow-Willis specification, Baqaee et al. (2023) suggested using empirical pass-through and sales share to back out the markup. Specifically, They characterized the theoretical relationship among markup, sales share, and pass-through as follows:

$$d\log\mu_i = (\mu_i - 1)\frac{1 - b_i}{b_i} d\log\lambda_i, \quad \text{s.t. } E_\lambda[\mu_i^{-1}]^{-1} = \bar{\mu}, \tag{C.1}$$

where μ_i represents firm i's markup, b_i is the pass-through, and λ_i is the sales share. Using the empirical relation between pass-through (b_i) and sales share (λ_i) of Belgian firms, Baqaee et al. (2023) back out μ_i from equation (C.1).

To assess the impact on our decomposition, we examine the relationship between empirical b_i and μ_i from equation (C.1), using the empirical pass-through of Amiti et al. (2019) and the sales share of Korean firms (λ_i) from Census data[28]. We then use the relationship between b_i and μ_i implied by equation (C.1) to impute firm-level pass-through (b_i), corresponding to its empirical markup (μ_i).

Table C.2 reports the decomposition results with aggregate elasticities computed with alternative pass-through. As expected, a higher dispersion in pass-through amplifies markup channel. With alternative pass-through, the aggregate markup rises 0.06 percentage points more while factor substitution is rarely affected. For equipment, an increase in markup is more pronounced: Markup channel reduces labor share 1.6 percentage points more than the baseline case, while between adjustment

[28] More specifically, Amiti et al. (2019) provides empirical b_i by a firm's employment size. We match Amiti et al. (2019)'s b_i with Korean λ_i, corresponding to the same employment density.

in factor substitution is slightly attenuated. Overall, capital-embodied technological changes (software and equipment combined) account for all the decline in the aggregate labor share observed in the data.

- Total changes in labor share : -0.044

- Effects of Software-embodied Technological Change ($\Delta \ln 1/q$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.034	-0.013	-0.015	+0.001	-0.021	-0.018	-0.003
(% explained)	(78.0)	(30.5)	(33.0)	(-2.5)	(47.5)	(40.1)	(7.4)

- Effects of Equipment-embodied Technological Change ($\Delta \ln 1/r$)

	Total	Factor Substitution			Markup		
		Overall	Within	Between	Overall	Within	Between
Changes	-0.014	+0.011	+0.014	-0.004	-0.025	-0.024	-0.001
(% explained)	(31.9)	(-24.2)	(-32.8)	(8.6)	(56.0)	(54.8)	(1.2)

Table C.2: Effects of Capital-Embodied Technological Change with Alternative Pass-Through

The decomposition is for the periods between 1990 and 2018. Percent explained of the overall labor share decline in the data in parentheses.

(a) Software Contribution (b) Equipment. Contribution

Fig. C.2: Effects of Capital-Embodied Technological Change with Alternative Pass-Through

The overall effect is calculated using the macro elasticity ($\bar{\sigma}$) and the within effect is computed based on the micro elasticity (σ) and the average pass-through ($\bar{b}$). The between effect is the difference between the effects based on macro elasticity and micro elasticity.

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